Temporal Many-valued Conditional Logics: an Abridged Report

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Abstract

In this paper we propose a many-valued temporal conditional logic. We start from a many-valued logic with typicality, and extend it with the temporal operators of the Linear Time Temporal Logic (LTL), thus providing a formalism which is able to capture the dynamics of a system, trough strict and defeasible temporal properties. We consider the many-valued case, while the two-valued case can be regarded as a special case.

Keywords

Preferential and Conditional reasoning, Temporal Reasoning, Typicality

1. Introduction

In this short paper we report about our work on a temporal extension of a many-valued conditional logic, based on a preferential approach to commonsense reasoning [1, 2, 3, 4, 5, 6]. The paper develops a *propositional many-valued temporal logic with typicality*, by extending the many-valued conditional logic with typicality introduced in [7] with temporal operators from the Linear Time Temporal Logic (LTL). This allows considering the temporal dimension, when reasoning about the defeasible typicality properties of a system, for explanation.

Preferential extensions of LTL with defeasible temporal operators have been recently studied [8, 9, 10] to enrich temporal formalisms with non-monotonic reasoning features, by considering defeasible versions of the LTL operators. Our approach, instead, adds the standard LTL operators to a (many-valued) conditional logic with typicality, an approach similar to the preferential extension considered for Description Logics (DLs), where the logic $LTL_{\mathcal{ALC}}$ [11], extending $\mathcal{ALC}$ with LTL operators, has been further extended with a *typicality operator*, in the two-valued [12] and many-valued case [13] to allow for conditional reasoning.

As in the Propositional Typicality Logic (PTL) by Booth et al. [14] (and in the DLs with typicality [15]) the conditionals are formalized based on material implication (resp., concept inclusions) plus the *typicality operator* $\mathbf{T}$. *Conditional implications* $\mathbf{T}(\alpha) \rightarrow \beta$, meaning that “normally if α holds, β holds", corresponds to conditionals $\alpha \mid\sim \beta$ in KLM logics [4, 6]. In this paper, as in [7], we further consider a many-valued semantics, so that a formula is given a value in a *truth degree set* $\mathcal{D}$, and the two-valued case can be regarded as a special case, obtained for $\mathcal{D} = \{0, 1\}$.

As the logic is many-valued, we consider *graded conditionals* of the form $(\mathbf{T}(\alpha) \rightarrow \beta) \geq l$, resp., $(\mathbf{T}(\alpha) \rightarrow \beta) \leq l$, meaning that “normally if α holds, then β holds, with degree at least (resp., at most) l" (in the following, we will omit the parentheses in $(\mathbf{T}(\alpha) \rightarrow \beta) \geq l$, and simply write $\mathbf{T}(\alpha) \rightarrow \beta \geq l$). For instance, the formalism allows for representing graded implications as: $living\_in\_Town \wedge Young \rightarrow \mathbf{T}(\Diamond Granted\_Loan) \geq l$, meaning that living in town and being young, implies that normally the loan is eventually granted, and the implication has degree at least l, where the interpretation of some propositions (e.g., $Young$) may be non-crisp.

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The preferential semantics of the logic exploits *multiple preference relations* $<_\alpha$ with respect to different formulas α, following the *multi-preferential semantics* developed for ranked and weighted DL knowledge bases (KBs) [16, 17], as well as for propositional conditionals, based on preferences with respect to different aspects [18]. The semantics considered in this paper generalized the approach in [18], which specifically deals with a multi-preferential extensions of the rational closure semantics.

The schedule of the paper is the following. Section 2 develops a many-valued preferential logic with typicality. Section 3 extends such logic with LTL modalities to develop a temporal many-valued conditional logic, and *temporal graded formulas*. Section 4 concludes the paper. An extended version of the paper, also dealing with weighted knowledge bases and gradual argumentation, can be found in [19].

2. A Many-valued Preferential Logics with Typicality

Let $\mathcal{L}$ be a propositional many-valued logic, whose formulas are built from a set $Prop$ of propositional variables using the logical connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$, as usual. We assume that $\bot$ and $\top$are formulas of $\mathcal{L}$. We consider a many-valued semantics for formulas, over a *truth degree set* $\mathcal{D}$, equipped with a preorder relation $\leq^{\mathcal{D}}$, a bottom element $0^{\mathcal{D}}$, and a top element $1^{\mathcal{D}}$. We denote by $<^{\mathcal{D}}$ and $\sim^{\mathcal{D}}$ the related strict preference relation and equivalence relation (often we will omit explicitly referring to $\mathcal{D}$).

Let $\otimes$, $\oplus$, $\ominus$ and $\rhd$ be the *truth degree functions* in $\mathcal{D}$ for the connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$ (respectively). When $\mathcal{D}$ is $[0, 1]$ or the finite truth space $\mathcal{C}_n = \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, \frac{n}{n}\}$, for an integer $n \geq 1$, as in our case of study [20], $\otimes$, $\oplus$, $\rhd$ and $\ominus$ can be chosen as a t-norm, an s-norm, an implication function, and a negation function in some system of many-valued logic [21]; for instance, in Gödel logic (that we will consider later): $a \otimes b = min\{a, b\}$, $a \oplus b = max\{a, b\}$, $a \rhd b = 1$ *if* $a \leq b$ *and* b *otherwise*; and $\ominus a = 1$ *if* $a = 0$ *and* 0 *otherwise*.

We further extend the language of $\mathcal{L}$ by adding a typicality operator as introduced by Booth et al. [14] for propositional calculus, and by Giordano et al. for preferential description logics [22]. Intuitively, "a sentence of the form $\mathbf{T}(A)$ is understood to refer to the *typical situations in which A holds*" [14]. The typicality operator allows the formulation of *conditional implications* (or *defeasible implications*) of the form $\mathbf{T}(A) \rightarrow B$ whose meaning is that "normally, if A then B", or "in the typical situations when A holds, B also holds". As in PTL [14], the typicality operator cannot be nested. When A and B do not contain occurrences of the typicality operator, an implication $A \rightarrow B$ is called *strict*. We call $\mathcal{L}^{\mathbf{T}}$ the language obtained by extending $\mathcal{L}$ with a unary typicality operator $\mathbf{T}$.

The interpretation of a typicality formula $\mathbf{T}(A)$ is defined with respect to a preferential interpretation. The KLM preferential semantics [4, 6, 3] exploits a set of worlds $\mathcal{W}$, with their valuation and a preference relation $<$ among worlds, to provide an interpretation of conditional formulas. Informally, a conditional $A \mathrel{|\!\sim} B$ is satisfied in a preferential interpretation, if B holds in all the most normal worlds satisfying A, i.e., in all $<$-minimal worlds satisfying A.

Here we consider a many-valued multi-preferential semantics for conditionals. The propositions at each world $w \in \mathcal{W}$ have a value in $\mathcal{D}$ and multiple preference relations $<_{A_i} \subseteq W \times W$ are associated to formulas A_i of $\mathcal{L}$. Multi-preferential semantics have been previously considered for defining refinements of the rational closure construction [23, 18], as well as for defeasible DLs, both in the two-valued and in the many-valued case, for *ranked KBs* [24, 25, 26].

Definition 1. *A* (multi-)preferential interpretation *is a triple* $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ *where:*

- *$\mathcal{W}$ is a non-empty set of worlds;*
- *each $<_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ is a strict partial order relation on $\mathcal{W}$;*
- *$v : \mathcal{W} \times Prop \longrightarrow \mathcal{D}$ is a valuation function, assigning a truth value in $\mathcal{D}$ to each propositional variable at each world $w \in \mathcal{W}$.*

The valuation v is inductively extended to all formulas in $\mathcal{L}^{\mathbf{T}}$:

$v(w, \bot) = 0_{\mathcal{D}}$ $\quad$ $v(w, \top) = 1_{\mathcal{D}}$ $\quad$ $v(w, A \rightarrow B) = v(w, A) \rhd v(w, B)$

$v(w, \neg A) = \ominus v(w, A)$ $\quad$ $v(w, A \wedge B) = v(w, A) \otimes v(w, B)$ $\quad$ $v(w, A \vee B) = v(w, A) \oplus v(w, B)$

$$v(w, \mathbf{T}(A)) = \begin{cases} v(w, A) & \text{if } \nexists w' \in \mathcal{W} \text{ s.t. } w' <_A w \\ 0_{\mathcal{D}} & \text{otherwise} \end{cases}$$

When $v(w, \mathbf{T}(A)) \neq 0_{\mathcal{D}}$, w is a typical/normal A-world in $\mathcal{M}$. Note that, differently from the KLM semantics [4, 6], we are not assuming well-foundedness of $<_A$.

Let us define the satisfiability in $\mathcal{M}$ of a *graded implication*, with form $A \rightarrow B \geq l$ or $A \rightarrow B \leq u$, where l and u are constants corresponding to truth values in $\mathcal{D}$ and A and B are formulas of $\mathcal{L}^{\mathbf{T}}$.

Given a preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$, the truth degree of an implication $A \rightarrow B$ in $\mathcal{M}$ is defined as follows:

$$(A \rightarrow B)^{\mathcal{M}} = inf_{w \in \mathcal{W}}(v(w, A) \rhd v(w, B)).$$

The satisfiability of a graded implication is evaluated globally to the preferential interpretation $\mathcal{M}$.

Definition 2. *A preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$, satisfies a graded implication $A \rightarrow B \geq l$ (written $\mathcal{M} \models A \rightarrow B \geq l$) iff $(A \rightarrow B)^{\mathcal{M}} \geq l$. Similarly, for $A \rightarrow B \leq u$.*

In general, some conditions may be needed to enforce an *agreement* between the truth values of a formula A_i at the different worlds in $\mathcal{M}$ and the preference relations $<_{A_i}$ among them. The preferences $<_{A_i}$ might have been determined by some *closure construction*, such as those exploiting the ranks or weights of conditionals in [24, 25]. Similar conditions, called coherence, faithfulness and φ-coherence conditions have, for instance, been introduced in the multi-preferential semantics for DLs with typicality in [25, 26].

A (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ is *coherent* if, for all $w, w' \in \mathcal{W}$, and preference relation $<_{A_i}$,

$$v(w, A_i) > v(w', A_i) \iff w <_{A_i} w'$$

that is, the ordering among the values of A in w and w' is justified by the preference relation $<_A$; and vice-versa. A weaker condition is faithfulness, only requires that $v(w, A_i) > v(w', A_i) \Rightarrow w <_{A_i} w'$.

Clearly, a preferential interpretation $\mathcal{M}$ might be coherent with respect to a preference relation $<_{A_i}$, while being only faithful with respect to another $<_{A_j}$.

We let a *knowledge base* K be a set of graded implications. A *model of* K is an interpretation $\mathcal{M}$ which satisfies all the graded implications in K. Given a knowledge base K, we say that K *entails* a graded implication $A \rightarrow B \geq l$ if $A \rightarrow B \geq l$ is satisfied in all the models of K (and similarly for $A \rightarrow B \leq l$). In the following, we will refer to the entailment of a graded implication $A \rightarrow B \geq 1$ as 1-*entailment*.

The KLM properties of a *preferential consequence relation* can be reformulated in the many-valued setting, and it can be proven that, for the choice of combination functions as in Gödel logic, they hold for 1-entailment, under the assumptions that $\mathcal{M}$ is coherent and the preference relations $<_{A_i}$ are well-founded.

Note that KLM preferential interpretations, with a single well-founded preference relation, can be regarded as a special case of multi-preferential interpretations. It can be proven that any KLM preferential interpretation [4] can be mapped into a two-valued multi-preferential interpretation satisfying the same conditionals.

3. A Temporal Preferential Logic with Typicality

In this section we extend the language of the logic $\mathcal{L}^{\mathbf{T}}$ with the temporal operators $\bigcirc$ (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of Linear Time Temporal Logic (LTL) [27].

First, we allow temporal operators and typicality operators to occur in a graded implication $A \rightarrow B \geq l$ (or $A \rightarrow B \geq l$) in A and in B, with the only restriction that $\mathbf{T}$ should not be nested. For instance, $lives\_in\_town \wedge young \rightarrow \mathbf{T}(\Diamond granted\_loan) \geq 0.8$ and $\Diamond \mathbf{T}(granted\_loan) \rightarrow lives\_in\_town \wedge young \geq 0.8$. are graded implication. Then, we will allow for combining graded implications.

The semantics of the many-valued temporal logic with typicality is defined in agreement with the semantics by Frigeri et al. [28].

Definition 3. *A* temporal (multi-)preferential interpretation *is a triple* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *where:*

- $\mathcal{W}$ *is a non-empty set of worlds;*
- *each* $<^n_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ *is partial order on* $\mathcal{W}$*;*
- $v : \mathbb{N} \times \mathcal{W} \times Prop \longrightarrow \mathcal{D}$ *is a valuation function assigning, at each time point, a truth value to any propositional variable in each world* $w \in \mathcal{W}$*.*

When there is no $w' \in \mathcal{W}$ s.t. $w' <^n_A w$, we say that w is a normal situation for A at timepoint n.

In a preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$, the valuation $v(n, w, A)$ of a formula A, in world w at time point $n \in \mathbb{N}$, can be defined inductively as follows:

$v(n, w, \bot) = 0_\mathcal{D}$ $\quad v(n, w, \top) = 1_\mathcal{D}$ $\quad v(n, w, \neg A) = \ominus v(n, w, A)$

$v(n, w, A \wedge B) = v(n, w, A) \otimes v(n, w, B)$ $\quad v(n, w, A \vee B) = v(n, w, A) \oplus v(n, w, B)$

$$v(n, w, \mathbf{T}(A)) = \begin{cases} v(n, w, A) & \text{if } \nexists w' \in \mathcal{W} \text{ s.t. } w' <^n_A w \\ 0_\mathcal{D} & \text{otherwise} \end{cases}$$

$v(n, w, \bigcirc A) = v(n + 1, w, A)$

$v(n, w, \Diamond A) = \bigoplus_{m\geq n} v(m, w, A)$ $\quad v(n, w, \Box A) = \bigotimes_{m\geq n} v(m, w, A)$

$v(n, w, A\mathcal{U}B) = \bigoplus_{m\geq n}(v(m, w, B)\otimes \bigotimes_{k=n}^{m-1} v(k, w, A))$

The semantics of $\Diamond$, $\Box$ and $\mathcal{U}$ requires a passage to the limit. Following [28], we introduce a bounded version for $\Diamond$, $\Box$ and $\mathcal{U}$, by adding new temporal operators $\Diamond_t$ (eventually in the next t time points), $\Box_t$ (always within t time points) and $\mathcal{U}_t$, with the interpretation:

$v(n, w, \Diamond_t A) = \bigoplus_{m=n}^{n+t} v(m, w, A)$ $\quad v(n, w, \Box_t A) = \bigotimes_{m=n}^{n+t} v(m, w, A)$

$v(n, w, A\mathcal{U}_t B) = \bigoplus_{m=n}^{n+t}(v(m, w, B)\otimes \bigotimes_{k=n}^{m-1} v(k, w, A))$

so that

$v(n, w, \Diamond A) = lim_{t\to+\infty}\, v(n, w, \Diamond_t A)$ $\quad v(n, w, \Box A) = lim_{t\to+\infty}\, v(n, w, \Box_t A)$

$v(n, w, A\mathcal{U}B) = lim_{t\to+\infty} v(n, w, A\mathcal{U}_t B)$.

Existence of the limits is ensured by the fact that $v(n, w, \Diamond_t C)$ and $v(n, w, C\mathcal{U}_t D)$ are increasing in t, and $v(n, w, \Box_t C)$ is decreasing in t (assuming usual properties of t-norms and t-conorms for $\otimes$ and $\oplus$).

As a consequence, for the case $\mathcal{D} = [0, 1]$, without the typicality operator, the semantics corresponds to the semantics of FLTL (Fuzzy Linear-time Temporal Logic) by Lamine and Kabanza [29], i.e.,

$v(n, w, \Diamond A) = v(n, w, A) \oplus v(n + 1, w, \Diamond A)$

$v(n, w, \Box A) = v(n, w, A) \otimes v(n + 1, w, \Box A)$

$v(n, w, A\mathcal{U}B) = v(n, w, B)\oplus (v(n, w, A) \otimes v(n + 1, w, A\mathcal{U}B))$.

Definition 4. *Given a temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *the* truth degree of an implication $A \to B$ in $\mathcal{I}$ at time point n *is:* $(A \to B)^{\mathcal{I},n} = inf_{w\in\mathcal{W}}(v(n, w, A) \rhd v(n, w, B))$.

Note that a temporal many-valued interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ can be regarded as a sequence of (non-temporal) preferential interpretations $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots$ where each $\mathcal{M}_n$ is defined as follows: $\mathcal{M}_n = \langle \mathcal{W}, \{<^n_{A_i}\}, v^n\rangle$, where $w <^n_{A_i} w'$ holds in $\mathcal{M}_n$ iff $w <^n_{A_i} w'$ holds in $\mathcal{I}$, for all $w, w' \in \mathcal{W}$; and $v^n(w, A) = v(n, w, A)$, for all $w \in \mathcal{W}$.

In the temporal case, rather than regarding graded implications as global constraints, that have to hold at all the time points, we allow for boolean combination of graded implications (as done in [7]) and also for the temporal operators to occur in front of the graded implications and of their boolean combinations. We call such formulas temporal graded formulas. A *temporal graded formula* is defined as follows:

$$\alpha ::= A \to B \geq l \mid A \to B \geq l \mid \alpha \wedge \beta \mid \neg\alpha \mid \bigcirc\alpha \mid \Diamond\alpha \mid \Box\alpha \mid \alpha\mathcal{U}\beta,$$

where α and β stand for temporal graded formulas. Note that temporal operators may occur both within graded implications ($A \to B \geq l$) and in front of them, and of their boolean combinations. An example of temporal graded formula is the following conjunction:

$\Box(\mathbf{T}(professor) \to teaches\,\mathcal{U}\; retired \geq 0.7) \wedge$

$(lives\_in\_town \wedge young \to \mathbf{T}(\Diamond granted\_loan) \geq 0.8)$

where the graded implication in the first conjunct is prefixed by a $\Box$ operator, while the second one is not.

A *temporal conditional KB* is a set of temporal graded formulas. We evaluate the satisfiability of a temporal graded formula at the initial time point 0 of a temporal preferential interpretation $\mathcal{I}$, essentially, as in LTL. Observe that any graded implication $A \rightarrow B \geq l$ is either satisfied or not at a time point n of a temporal interpretation $\mathcal{I}$, i.e., either $\mathcal{I}, n \models A \rightarrow B \geq l$ or $\mathcal{I}, n \not\models A \rightarrow B \geq l$ (and similarly for the graded implications with $\leq$). Hence, the interpretation above of temporal graded formulas in $\mathcal{I}$ at a time point n is two-valued (although it builds over the degree of an implication $A \rightarrow B$ in $\mathcal{I}$ at time point n, which has a truth value $(A \rightarrow B)^{\mathcal{I},n}$ in $\mathcal{D}$). We refer to [19] for the detailed definition of satisfiability of a temporal graded formula at time point n, and define the notions of satisfiability and entailment as follows.

Definition 5 (Satisfiability and entailment)**.** *A temporal graded formula α is* satisfied *in a temporal preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ if $\mathcal{I}, 0 \models \alpha$.*

A preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ is a model *of a temporal conditional knowledge base K, if $\mathcal{I}$ satisfies all the temporal graded formulas in K.*

A temporal conditional knowledge base K entails *a temporal graded formula α if α is satisfied in all the models $\mathcal{I}$ of K.*

Note that, in the temporal graded formula given above, the graded implication in the first conjunct ($\mathbf{T}(professor) \rightarrow teaches\,\mathcal{U}\ retired \geq 0.7$) is required to hold at all the time points of the interpretation $\mathcal{I}$ (as it is prefixed by $\Box$), while the second conjunct ($lives\_in\_town \wedge young \rightarrow \mathbf{T}(\Diamond granted\_loan) \geq 0.8$) has to hold only at time point 0.

Decidability and complexity of the different decision problems (the satisfiability, the model checking and entailment problems) have to be studied for this temporal many-valued conditional logic, for different choices of $\mathcal{D}$ and of combination functions. In the two-valued case, a related formalism which extends the temporal description logic $LTL_{\mathcal{ALC}}$ [11] with the typicality operator, has been shown to be decidable when only a finite set of preference relations $<_{A_i}$ is considered [12], and concept inclusions are regarded as global temporal constraints.

As in the two-valued non-temporal case, the notion of preferential entailment considered in this section is rather weak. For the KLM logics, some different closure constructions have been proposed to strengthen entailment by restricting to a subset of the preferential models of a conditional knowledge base K. Let us just mention, the rational closure [6], the lexicographic closure [30], and the MP-closure [18].

In this direction, we consider *weighted temporal KBs*, which allow defeasible implications with a weight, following an approach first proposed for weighted KBs in defeasible DLs [26, 13]. A *weighted KB* is a set of *weighted typicality implication* of the form $(\mathbf{T}(A_i) \rightarrow B_j, w_{ij})$, where A_i and B_j are propositions, and the weight w_{ij} is a real number, representing the plausibility or implausibility of the conditional implication. For instance, for a proposition $student$, we may have a set of weighted defeasible implications:

($\mathbf{T}(student) \rightarrow has\_Classes$, +50), ($\mathbf{T}(student) \rightarrow \Diamond holds\_Degree$,+30) ,
($\mathbf{T}(student) \rightarrow has\_Boss$, -40),

that represent *prototypical properties* of students, i.e., that a student normally has classes and will eventually reach the degree, but she usually does not have a boss (negative weight). Accordingly, a student having classes, but not a boss, is more typical than a student having classes and a boss.

A weighted (defeasible) knowledge base K_D can coexist with a strict knowledge base K_S (i.e., a set of graded implications), as usual in defeasible DLs. We refer to the extended version [19] for a semantics of weighted temporal KBs and for an instantiation of the approach for gradual argumentation.

4. Conclusions

The paper proposes a framework in which different (many-valued) preferential logics with typicality can be captured, together with their temporal extensions, with the operators from LTL. The interpretation of the typicality operator is based on a multi-preferential semantics, and an extension of weighted conditional knowledge bases to the temporal (many-valued) case is suggested. In [19] we also consider an instantiation of the formalism to the verification of temporal properties of gradual argumentation graphs, an approach which extends the (multi-)preferential (typicality-based) approach for the verification of conditional properties of argumentation graphs in gradual argumentation semantics proposed in [7].

On a different route, in the two-valued case, a preferential logics with defeasible LTL operators has been studied in [9, 31]. The decidability of different fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [8, 31]. Our approach does not consider defeasible temporal operators nor preferences over time points, but combines standard LTL operators with the typicality operator in a many-valued temporal logic. We have not considered the additional temporal operators ("soon", "almost always", etc.) introduced by Frigeri et al. [28] for representing vagueness in the temporal dimension, they can be considered for future work. The quantitative value of satisfaction of extended LTL formulas is also considered in [32]. Our approach, besides being many-valued, exploit a typicality operator, which allows for conditional implications and makes the logic non-monotonic [4].

Future work also includes studying the decidability for fragments of the logic, developing proof methods, as in the non-temporal case [20, 33], exploiting the formalism for explainability, and for reasoning about the dynamics of argumentation graphs in a gradual semantics.

While conditional weighted KBs have been shown to capture the stationary states of some neural networks (or their finite approximation) [25, 26], and allow for combining empirical knowledge with elicited knowledge for post-hoc verification, adding a temporal dimension opens to the possibility of verifying properties concerning the dynamic behavior of a network.

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