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Reasoning in Multi-Agent Conformant Planning over Transition Systems*

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Abstract

Reasoning about actions and information is one of the most active areas of research in artificial intelligence. In this paper, we study the reasoning in multi-agent conformant planning. We introduce a formalism to trace the update of information in multi-agent conformant planning over transition systems. We propose a dynamic epistemic logical framework $\texttt{EAL}^n$ to capture reasoning in such scenarios and present an upper bound on the time complexity of multi-agent conformant planning in terms of $\texttt{EAL}^n$.

1 Introduction

Conformant planning, which is a branch of artificial intelligence (AI), deals with devising an action sequence to achieve a goal in the presence of uncertainty about the initial state (see [15]). Tracking the update of the uncertainty during the execution of actions plays an important role. In single-agent settings, the update of uncertainty is direct and intuitive. However, the situation becomes much more involved in multi-agent settings, since nested beliefs need to be considered. How to intuitively represent and track the update of uncertainty in multi-agent planning is one of the main challenges in the AI community.

Instead of using transition systems as the underlying formalism of planning, in recent years, there has been a growing interest in handling multi-agent planning in dynamic epistemic logic (DEL) framework (see e.g., [5, 12, 2, 3, 17, 14, 10, 6, 13]). Usually, it is called *epistemic planning* in literature. In epistemic planning, states are epistemic models, actions are event models and the state transitions are implicitly encoded by the update product which computes a new epistemic model based on an epistemic model and an event model. One advantage of this approach is its expressiveness in handling partially observable actions, such as private announcements. However, this expressiveness comes at a price, as shown in [5, 4], multi-agent epistemic planning is undecidable in general. Moreover, if only fully observable actions are considered, the DEL formalism of event models is more complex and less intuitive than transition systems. In [11, 16], another approach is proposed which uses the core-idea of DEL but not its formalism to track the uncertainty change over transition systems in single-agent conformant planning, but this approach cannot be applied to handle multi-agent planning.

This paper proposes a semantic-driven dynamic epistemic framework for reasoning about knowledge and action in multi-agent conformant planning. The main contributions of this paper are summarized as follows: 1) We introduce an update formalism to track information change in multi-agent conformant

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planning over transition systems. The update formalism can be seen as a generalized version of the approach introduced in [11], but it is not a directly generalized result. In [11], the agent's belief is represented by a set of states that the agent cannot distinguish. This representation does not work anymore in multi-agent settings, since nested beliefs need to be considered. In this paper, we use a non-well-founded notion, *possibility*, to represent belief states in multi-agent planning, and we define the update on possibilities to capture information change. 2) We propose a logical framework $\mathtt{EAL}^n$ with DEL-style update semantics and expressive language. In the literature of epistemic planning, such as [6], the logical language is an extension of proposition logic with knowledge modalities. The language of $\mathtt{EAL}^n$ has not only knowledge modalities but also action modalities, which allows us to express more complicated planning goals. Moreover, we can talk about the interaction between action and knowledge in the logical language. It would give us a more in-depth understanding of reasoning about action and information in multi-agent planning. 3) We present an upper bound on the time complexity of multi-agent conformant planning in terms of $\mathtt{EAL}^n$. We show that multi-agent conformant planning with goal expressed by an $\mathtt{EAL}^n$-formula is in double exponential time.

2 Information change in multi-agent conformant planning

Let $\mathbf{Act}$ be a set of actions, $\mathbf{P}$ be a set of proposition variables, and $\mathbf{Ag}$ be a set of agents.

A *transition system* $\mathcal{T}$ is a triple $\langle S^{\mathcal{T}}, \{R_a^{\mathcal{T}} \mid a \in \mathbf{Act}\}, V^{\mathcal{T}}\rangle$, where $S^{\mathcal{T}}$ is a non-empty set of states, for each $a \in \mathbf{Act}$, $R_a^{\mathcal{T}}$ is a function from S to $\mathcal{P}(S)$, and $V^{\mathcal{T}} : \mathbf{P} \to \mathcal{P}(S^{\mathcal{T}})$ is an assignment function.

We use possibility on transition systems, which is a variant of the notion used in [8, 7], to represent agents' epistemic uncertainty about states. Before introducing the notion of possibility, let us start by introducing some auxiliary notions.

Let $\mathcal{T}$ be a transition system. A *Kripke structure* $\mathcal{N}$ on $\mathcal{T}$ is a triple $\langle W^{\mathcal{N}}, \{\overset{i}{\dashrightarrow} \mid i \in \mathbf{Ag}\}, L^{\mathcal{N}}\rangle$, where $W^{\mathcal{N}}$ is a non-emptyset of possible worlds, for each $i \in \mathbf{Ag}$, $\overset{i}{\dashrightarrow}$ is a binary relation on W, and $L^{\mathcal{N}} : W^{\mathcal{N}} \to S^{\mathcal{T}}$ is a function that labels each possible world with a state in $\mathcal{T}$. For each $w \in W^{\mathcal{N}}$, we say $(\mathcal{N}, w)$ is a *pointed Kripke structure*.

A Kripke structure $\mathcal{N}$ on $\mathcal{T}$ models agents' uncertainty over states in $\mathcal{T}$. By [1], we know that a pointed Kripke structure can be represented by the following notion from non-well-founded set theory.

Let $\mathcal{N}$ be a Kripke structure on a transition system $\mathcal{T}$. A *decoration* d of $\mathcal{N}$ is a function that assigns to each world $w \in W$ a function $d(w)$ such that

- $d(w)(\mathcal{T}) = L^{\mathcal{N}}(w)$ (i.e., $d(w)$ assigns to $\mathcal{T}$ a state that is the one with which L labels w), and
- for each $i \in \mathbf{Ag}$, $d(w)(i) = \{d(w') \mid w \overset{i}{\dashrightarrow} w'\}$ (i.e., $d(w)$ assigns to each agent i the (non-well-founded) set of functions associated with worlds reachable from w by $\overset{i}{\dashrightarrow}$).

If d is a decoration of $\mathcal{N}$ and w is a world in $\mathcal{N}$, we say that $d(w)$ is the solution of w in $\mathcal{N}$, and $(\mathcal{N}, w)$ is a picture of $d(w)$.

Now we are ready to introduce the concept of possibility.

Definition 1 (Possibility)**.** *Let $\mathcal{T}$ be a transition system. A* possibility α *on $\mathcal{T}$ is a function that assigns to $\mathcal{T}$ a state in $S^{\mathcal{T}}$, and to each agent $i \in \mathbf{Ag}$ a set of possibilities.*

As shown in [8], there is a close relationship between these (non-well-founded) possibilities on $\mathcal{T}$ and Kripke structures on $\mathcal{T}$. Each Kripke structure $\mathcal{N}$ has a unique decoration that assigns to each world in $\mathcal{N}$ a possibility, and each possibility has a picture. For example, let $\mathcal{T}_0$ be the transition system depicted in Figure 1. A possibility α_1 on $\mathcal{T}_0$ is presented in Figure 2a. The pointed Kripke structure $(\mathcal{N}, w_1)$ on $\mathcal{T}_0$, depicted in Figure 2b, is a picture of α_1.

The choice of possibilities over Kripke structures as uncertainty representation provides several advantages (see [9]). One of them is that update on possibility is natural and elegant. The update on possibilities, defined in the following, tracks the change of agents' uncertainty when an action is executed.

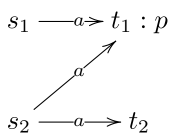
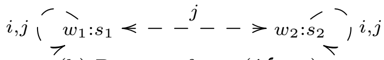


Figure 1: Transition system $\mathcal{T}_0$

$$\begin{cases} \alpha_1 = & \{(i,(\alpha_1,\alpha_1)),(j,(\alpha_1,\alpha_1)),\\ & (j,(\alpha_1,\alpha_2)),s_1\}\\ \alpha_2 = & \{(i,(\alpha_2,\alpha_2)),(j,(\alpha_2,\alpha_1)),\\ & (j,(\alpha_2,\alpha_2)),s_2\}\end{cases}$$

(a) System of equations of α_1



(b) Picture of α_1: $(\mathcal{N}, w_1)$

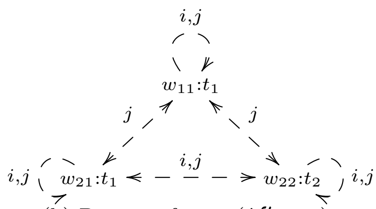
Figure 2: Possibility α_1 on $\mathcal{T}_0$

Definition 2 (Update)**.** *Let α be a possibility on $\mathcal{T}$. The* update *of α with an action $a \in \mathbf{Act}$, denoted $\alpha|^a$, is a set of possibilities on $\mathcal{T}$ such that $\alpha' \in \alpha|^a$ if and only if $\alpha'(\mathcal{T}) \in R_a(\alpha(\mathcal{T}))$ and $\alpha'(i) = \bigcup_{\beta\in\alpha(i)} \beta|^a$.*

As an example, the update of the possibility α_1 depicted in Figure 2a will result in a unit possibility set α_{11}, which is depicted in Figure 3. Moreover, the update of α_2 in Figure 2a, whose picture is $(\mathcal{N}, w_2)$ in Figure 2b, is the set of possibilities α_{21} and α_{22}, both of which are depicted in Figure 3. The pointed Kripke structures $(\mathcal{N}', w_{21})$ and $(\mathcal{N}', w_{22})$ are the pictures of α_{21} and α_{22} respectively. Moreover, we would like to point out that the difference between $\alpha_{11}(i)$ and $\alpha_{21}(i)$ corresponds to the fact that the information state of agent i at state t_1 is different depending on how he/she gets there, and the identity between $\alpha_{11}(j)$ and $\alpha_{21}(j)$ reflects the fact that the information state of agent j at t_1 is the same because he/she intuitively cannot distinguish s_1 and s_2.

$$\begin{cases} \alpha_{11} = & \{(i,(\alpha_{11},\alpha_{11})),\\ & (j,(\alpha_{11},\alpha_{11})),(j,(\alpha_{11},\alpha_{21})),(j,(\alpha_{11},\alpha_{22})),t_1\}\\ \alpha_{21} = & \{(i,(\alpha_{21},\alpha_{21})),(i,(\alpha_{21},\alpha_{22})),\\ & (j,(\alpha_{21},\alpha_{11})),(j,(\alpha_{21},\alpha_{21})),(j,(\alpha_{21},\alpha_{22})),t_1\}\\ \alpha_{22} = & \{(i,(\alpha_{22},\alpha_{21})),(i,(\alpha_{22},\alpha_{22})),\\ & (j,(\alpha_{22},\alpha_{11})),(j,(\alpha_{22},\alpha_{21})),(j,(\alpha_{22},\alpha_{22})),t_2\}\end{cases}$$

(a) System of equations of α_{11}



(b) Picture of α_{11}: $(\mathcal{N}', w_{11})$

Figure 3: Possibility α_{11} on $\mathcal{T}_0$

3 The logic $\texttt{EAL}^n$ and its application

In this section, we will firstly introduce the language and the semantics of $\texttt{EAL}^n$ and then use it to capture reasoning in multi-agent conformant planning. At the end of the section, we provide an upper bound for multi-agent conformant planning in terms of $\texttt{EAL}^n$.

Definition 3 (Language)**.** *The language $\mathcal{L}_{\texttt{EAL}^n}$ is defined by the following BNF:*

$$\phi ::= p \mid \neg\phi \mid \phi\wedge\phi \mid [a]\phi \mid \mathsf{K}_i\phi,$$

where $p \in \mathbf{P}$, $a \in \mathbf{Act}$, and $i \in \mathbf{Ag}$. We use $\bot, \vee, \to, \langle\cdot\rangle$ as the usual abbreviations.

The formula $[a]\phi$ intuitively means that ϕ holds if the action a is successfully done. The formula $\mathsf{K}_i\phi$ expresses that the agent i knows that ϕ. Since the language of $\texttt{EAL}^n$ contains both action modalities and knowledge modalities, this allows one to talk about the interaction between action and knowledge in the object language.

Formulas of $\mathcal{L}_{\texttt{EAL}^n}$ will be interpreted on dynamic epistemic models. A *dynamic epistemic model* is a pair $(\mathcal{T}, \alpha)$, where $\mathcal{T}$ is a transition system and α is a possibility on $\mathcal{T}$ such that for each $i \in \mathbf{Ag}$, α is in $\alpha(i)$ and $\alpha' \in \alpha(i)$ implies $\alpha'(i) = \alpha(i)$.

Definition 4 (Update semantics)**.** *Given a dynamic epistemic model* $(\mathcal{T}, \alpha)$ *and a formula* $\phi \in \mathcal{L}_{\texttt{EAL}^n}$*, the satisfaction relation* $\vDash$ *is defined as follows:*

$$\begin{aligned}
\mathcal{T}, \alpha \vDash p &\iff \alpha(\mathcal{T}) \in V(p)\\
\mathcal{T}, \alpha \vDash \neg\phi &\iff \mathcal{T}, \alpha \nvDash \phi\\
\mathcal{T}, \alpha \vDash \phi \wedge \psi &\iff \mathcal{T}, \alpha \vDash \phi \text{ and } \mathcal{T}, \alpha \vDash \psi\\
\mathcal{T}, \alpha \vDash \mathsf{K}_i\phi &\iff \text{for all } \alpha' \in \alpha(i) : \mathcal{T}, \alpha' \vDash \phi\\
\mathcal{T}, \alpha \vDash [a]\phi &\iff \text{for all } \alpha' \in \alpha|^a : \mathcal{T}, \alpha' \vDash \phi
\end{aligned}$$

We say that ϕ *is valid, denoted as* $\vDash \phi$*, if* $\mathcal{T}, \alpha \vDash \phi$ *for each dynamic epistemic model* $\mathcal{T}, \alpha$*.*

Within $\texttt{EAL}^n$, we can express the epistemic state of agents and its evolvement in multi-agent planning. Take the following example. Let $\mathcal{T}_0$ and α_1 be depicted as Figure 1 and Figure 2 respectively. We then have the following statements.

- $\mathcal{T}_0, \alpha_1 \vDash [a](\mathsf{K}_i p \wedge \neg\mathsf{K}_j p)$. When the initial possibility is α_1, after the action a is executed, agent i knows that p, and agent j does not know that p.
- $\mathcal{T}_0, \alpha_2 \vDash [a](\neg\mathsf{K}_i p \wedge \neg\mathsf{K}_j p)$. When the initial possibility is α_2, after the action a is executed, neither i not j knows that p.
- For all $\phi \in \mathcal{L}_{\texttt{EAL}^n}$, $\mathcal{T}_0, \alpha_1 \vDash [a]\mathsf{K}_j\phi$ iff $\mathcal{T}_0, \alpha_2 \vDash [a]\mathsf{K}_j\phi$. No matter the initial possibility is α_1 or α_2, after the action a is executed, the knowledge of j is the same.

Within $\texttt{EAL}^n$, we can capture reasoning about action and information change in multi-agent conformant planning.

Lemma 1 (Perfect Recall)**.** *For each dynamic epistemic model* $(\mathcal{T}, \alpha)$*, we have that* $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi \to [a]\mathsf{K}_i\phi$ *for each* $i \in \mathbf{Ag}$ *and each* $a \in \mathbf{Act}$*.*

Proof. Assume that $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$. We will show that $\mathcal{T}, \alpha \vDash [a]\mathsf{K}_i\phi$. Taka any $\alpha' \in \alpha|^a$ and any $\beta' \in \alpha'(i)$. To show that $\mathcal{T}, \alpha \vDash [a]\mathsf{K}_i\phi$, by the update semantics, we only need to show that $\mathcal{T}, \beta' \vDash \phi$.

Since $\alpha' \in \alpha|^a$, by Definition 2, this follows that $\alpha'(i) = \bigcup_{\beta\in\alpha(i)} \beta|^a$. Since $\beta' \in \alpha'(i)$, we then have that there is some $\beta \in \alpha(i)$ such that $\beta' \in \beta|^a$. Since $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$ and $\beta \in \alpha(i)$, this follows that $\mathcal{T}, \beta \vDash [a]\phi$. Moreover, since $\beta' \in \beta|^a$, this follows that $\mathcal{T}, \beta' \vDash \phi$. Therefore, we have shown that $\mathcal{T}, \beta' \vDash \phi$ for each $\alpha' \in \alpha|^a$ and each $\beta' \in \alpha'(i)$, namely $\mathcal{T}, \alpha \vDash [a]\mathsf{K}_i\phi$. Thus, we have shown that if $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$ then $\mathcal{T}, \alpha \vDash [a]\mathsf{K}_i\phi$. □

Lemma 2 (No Miracles)**.** *For each dynamic epistemic model* $(\mathcal{T}, \alpha)$*, we have that* $\mathcal{T}, \alpha \vDash \langle a\rangle\mathsf{K}_i\phi \to \mathsf{K}_i[a]\phi$ *for each* $i \in \mathbf{Ag}$ *and each* $a \in \mathbf{Act}$*.*

Proof. Assume that $\mathcal{T}, \alpha \vDash \langle a\rangle\mathsf{K}_i\phi$. We will show that $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$. By the update semantics, we only need to show that $\mathcal{T}, \beta' \vDash \phi$ for each $\alpha' \in \alpha(i)$ and each $\beta' \in \alpha'|^a$.

Since $\mathcal{T}, \alpha \vDash \langle a\rangle\mathsf{K}_i\phi$, this follows that there is some $\gamma \in \alpha|^a$ such that $\mathcal{T}, \gamma \vDash \mathsf{K}_i\phi$. Since $\gamma \in \alpha|^a$, by Definition 2, this follows that $\gamma(i) = \bigcup_{\beta\in\alpha(i)} \beta|^a$. Take any $\alpha' \in \alpha(i)$. We then have that $\alpha'|^a \subseteq \gamma(i)$. Take any $\beta' \in \alpha'|^a$. We then have that $\beta' \in \gamma(i)$. Since $\mathcal{T}, \gamma \vDash \mathsf{K}_i\phi$, this follows that $\mathcal{T}, \beta' \vDash \phi$. Thus, we have shown that $\mathcal{T}, \beta' \vDash \phi$ for each $\alpha' \in \alpha(i)$ and each $\beta' \in \alpha'|^a$, namely $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$. Thus, we have shown that if $\mathcal{T}, \alpha \vDash \langle a\rangle\mathsf{K}_i\phi$ then $\mathcal{T}, \alpha \vDash \mathsf{K}_i[a]\phi$. □

Intuitively, perfect recall means that if the agent cannot distinguish two states after doing action a, then he/she could not distinguish them before. No miracles mean that if the agent cannot distinguish two states and the same action is executed on both states, then he/she cannot distinguish the resulting states. These can be depicted in Figure 4.

We now introduce multi-agent conformant planning in terms of $\texttt{EAL}^n$. First, we introduce some auxiliary notations. Let Σ be a set of possibilities. We use $\Sigma|^a$ to denote the set $\bigcup_{\alpha\in\Sigma} \alpha|^a$. Let $\sigma \in \mathbf{Act}^*$ be an action sequence $a_1 \cdots a_n$. We use $\alpha|^\sigma$ to denote the information state $(\cdots((\alpha|^{a_1})|^{a_2})\cdots)|^{a_n}$.

Definition 5 (Multi-agent conformant planning)**.** *A* multi-agent conformant planning *problem P is a tuple* $\langle \mathcal{T}, \alpha, G, \{\mathbf{Act}_i \mid i \in G\}, \phi_g\rangle$*, where* $\mathcal{T}$ *is a transition system,* α *is the initial possibility on* $\mathcal{T}$*, G*

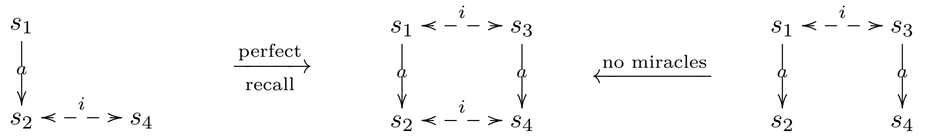


Figure 4: perfect recall and no miracles

is a group of agents, $\mathbf{Act}_i \subseteq \mathbf{Act}$ *is a set of actions that the agent* i *can perform,* $\phi_g \in \mathcal{L}_{\mathtt{EAL}^n}$ *is the goal formula. A* solution *to* P *is a finite sequence of actions* $\sigma = a_1 \cdots a_n \in (\bigcup_{i \in G} \mathbf{Act}_i)^*$ *such that 1) the sequence* σ *is applicable* [1] *in the state* $\alpha'(\mathcal{T})$ *for each* $\alpha' \in \bigcap_{i \in G} \alpha(i)$*, and 2)* $\mathcal{T}, \beta \vDash \phi_g$ *for each* $\alpha' \in \bigcap_{i \in G} \alpha(i)$ *and each* $\beta \in \alpha'|^{\sigma}$*.*

Let $(\!|a|\!)\phi$ be an abbreviation of the formula $\langle a \rangle \top \wedge [a]\phi$. It can be shown that an action sequence $a_1 \cdots a_n$ is applicable on the state $\alpha(\mathcal{T})$ iff $\mathcal{T}, \alpha \vDash (\!|a_1|\!) \cdots (\!|a_n|\!)\top$. We then have the following theorem, which means that the solution can be verified in $\mathtt{EAL}^n$.

Theorem 3. *Let* $P = \langle \mathcal{T}, \alpha, G, \{\mathbf{Act}_i \mid i \in G\}, \phi_g \rangle$ *be a planning problem. An action sequence* $a_1 \cdots a_n$ *is a solution for* P *iff* $\mathcal{T}, \alpha' \vDash (\!|a_1|\!) \cdots (\!|a_n|\!)\phi_g$ *for each* $\alpha' \in \bigcap_{i \in G} \alpha(i)$*.*

In this paper's remainder, we show an upper bound on the time complexity of multi-agent conformant planning in terms of $\mathtt{EAL}^n$.

Let $\mathcal{T} = \langle S, \{R_a \mid a \in \mathbf{Act}\}, V \rangle$ be a transition system and $\mathcal{N} = \langle W, \{\overset{i}{\dashrightarrow} \mid i \in \mathbf{Ag}\}, L \rangle$ be a Kripke structure on $\mathcal{T}$. The Kripke structure $\mathcal{N}^\dagger = \langle W^{\mathcal{N}^\dagger}, \{\overset{i}{\dashrightarrow}_{\mathcal{N}^\dagger} \mid i \in \mathbf{Ag}\}, L^{\mathcal{N}^\dagger} \rangle$ is defined as follows: $W^{\mathcal{N}^\dagger} = \{(w, L(w)) \mid w \in W\}$, $\overset{i}{\dashrightarrow}_{\mathcal{N}^\dagger} = \{((w, L(w)), (w', L(w'))) \mid w \overset{i}{\dashrightarrow} w'\}$, and $L^{\mathcal{N}^\dagger}(w, L(w)) = L(w)$. If $\mathcal{N}$ is a Kripke structure on $\mathcal{T}$, so is $\mathcal{N}^\dagger$. It is obvious that $(\mathcal{N}, w)$ is isomorphic to $(\mathcal{N}^\dagger, (w, L(w)))$. Therefore, if $(\mathcal{N}, w)$ is a picture of a possibility α, so is $(\mathcal{N}^\dagger, (w, L(w)))$.

Given a transition system $\mathcal{T}$ and a Kripke structure $\mathcal{N}$ on $\mathcal{T}$, let the Kripke structure $\mathcal{N}^\dagger$ is defined as above. We now define the update of $\mathcal{N}^\dagger$ with an action $a \in \mathbf{Act}$, denoted as $\mathcal{N}^\dagger|^a$. The Kripke structure $\mathcal{N}^\dagger|^a = \langle W^{\mathcal{N}^\dagger|^a}, \{\overset{i}{\dashrightarrow}_{\mathcal{N}^\dagger|^a} \mid i \in \mathbf{Ag}\}, L^{\mathcal{N}^\dagger|^a} \rangle$ is defined as follows: $W^{\mathcal{N}^\dagger|^a} = \{(w, s') \in W \times S \mid$ there is $(w, s) \in W^{\mathcal{N}^\dagger}$ such that $(s, s') \in R_a^{\mathcal{T}}\}$, $\overset{i}{\dashrightarrow}_{\mathcal{N}^\dagger|^a} = \{((w, s), (w', s')) \mid w \overset{i}{\dashrightarrow} w'\}$, and $L^{\mathcal{N}^\dagger|^a}(w, s) = s$. It is evident that $\mathcal{N}^\dagger|^a$ is also a Kripke structure on $\mathcal{T}$. We have known that if $(\mathcal{N}, w)$ is a picture of a possibility α, so is $(\mathcal{N}^\dagger, (w, L(w)))$. If $\alpha' \in \alpha|^a$, this follows that $\alpha(\mathcal{T}) R_a^{\mathcal{T}} \alpha'(\mathcal{T})$. Let $\alpha(\mathcal{T}) = s$ and $\alpha'(\mathcal{T}) = s'$. Since $(\mathcal{N}, w)$ is a picture of a possibility α, this follows that $L(w) = s$, and then we have that $(w, s) \in W^{\mathcal{N}^\dagger}$. Since $s R_a^{\mathcal{T}} s'$, by the definition of $\mathcal{N}^\dagger|^a$, we know that $(w, s') \in W^{\mathcal{N}^\dagger|^a}$. With the bisimulation principle (see [8]), it can be shown that $(\mathcal{N}^\dagger|^a, (w, s'))$ is a picture of α'. Moreover, it can be shown that for each action sequence $a_1 \cdots a_n$ and each $\beta \in \alpha|^{a_1} \cdots |^{a_n}$, the pointed Kripke structure $\mathcal{N}^\dagger|^{a_1} \cdots |^{a_n}, (w, \beta(\mathcal{T}))$ is picture of β. Since $W^{\mathcal{N}^\dagger|^{a_1} \cdots |^{a_n}} \subseteq W \times S$, this follows that the number of possibilities that can be generated by the initial possibility α is bounded by the size of the powerset of $W \times S$.

Given a possibility α, let the size of α be the size of the minimal pointed Kripke structure $(\mathcal{N}, w)$ such that $(\mathcal{N}, w)$ is a picture of α. Given a multi-agent conformant planning problem $P = \langle \mathcal{T}, \alpha, G, \{\mathbf{Act}_i \mid i \in G\}, \phi_g \rangle$, let the size of P be the product of the size of $\mathcal{T}$, the size of α, and the size of ϕ_g.

Theorem 4. *Multi-agent conformant planning in terms of* $\mathtt{EAL}^n$ *is in double exponential time.*

Proof idea. Given a multi-agent conformant planning P, we can do a *breadth-first search* with duplicate checking on possibilities to find a plan. Since we have shown above that the number of possibilities that can be generated by the initial possibility α is bounded by the exponent of the size of P, this follows that the depth of the searching tree is bounded by the exponent of the size of P. Therefore, the time of the breadth-first search is in double exponent of the size of P. Moreover, by Theorem 3, we know that the verification of a plan can be reduced to model checking in $\mathtt{EAL}^n$. Since model checking in $\mathtt{EAL}^n$ is in polynomial time, this follows that searching a plan for P is in double exponential time. ☐

[1] $a_1 \cdots a_n$ is applicable on a state s iff $R_{a_1}(s) \neq \emptyset$, and for each $1 \leq k < n$, $t \in R_{a_1 \cdots a_k}(s)$ implies $R_{a_{k+1}}(t) \neq \emptyset$.

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