Timeline-based Planning:Theory and Practice
Planning Domains and Non-Flexible Timelines

Dottorato in Informatica e Scienze Matematiche e Fisiche

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Context & Motivations

- Planning for real world problems with explicit temporal constraints is a challenging problem
- Flexible timeline-based Planning and Scheduling $(\mathrm{P\&S})$ has demonstrated to be successful in a number of concrete applications
- A remarkable research effort has been dedicated to design, build and deploy timeline-based software environments
- Nevertheless, a formal characterization of flexible timelines and flexible plans was missing
- A rather limited community put efforts to make research in this area

A Running Example: the Satellite Planning Domain

- A generic Planning problem in space domain
- A remote satellite is controlled by a planner and an executive system to accomplish some required tasks
- The satellite can either point to
  - a remote planet using its instruments to produce scientific data
  - an Earth ground communication station downlinking the scientific data previously stored
- A set of operative constraints are to be satisfied:
  - Point the planet to allow observations
  - Point Earth to transmit data to ground station
  - Communicate only if ground station is visible
  - Perform some maintenance operations

Planning Domains

Timeline based planning: synthesis of desired temporal behaviors (timelines) of time varying features (state variables)

The domain specification contains causal laws and constraints that must be obeyed:

- allowed value transitions
- durations of valued intervals
- constraints (synchronization) between different state variables
- information on intervals controllability/uncontrollability

The desired temporal behaviour includes satisfaction of given goals

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In general, time instants and durations are elements of an infinite set of non negative numbers $\mathbb{T}$, including 0

For instance, $\mathbb{T} = \mathbb{N}$ (discrete time framework), or $\mathbb{T} = \mathbb{R}_{\geq 0}$, the non-negative real numbers

$$\mathbb{T}^{\infty} = \mathbb{T} \cup \{\infty\}$$

State Variables

$$x = (V, T, \gamma, D)$$

- x: variable name
- V: set of values
- $T : V \to 2^V$: value transition function
- $\gamma : V \to \{c, u\}$: controllability tagging function

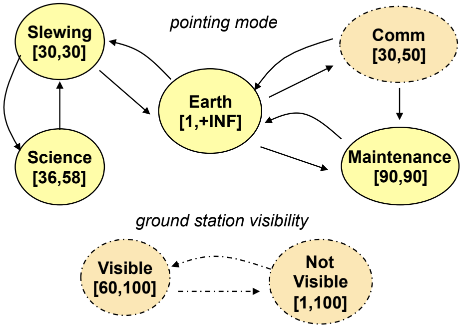
  $\gamma(v)$: controllability tag of the value v.

  $\gamma(v) = u$: v is an uncontrollable value
  (the controller cannot decide its exact duration)

  $\gamma(v) = c$: v is controllable
- $D : V \to \mathbb{T} \times \mathbb{T}^{\infty}$: duration function ; $D(v) = (t_{min}, t_{max})$, with $0 \leq t_{min} \leq t_{max}$

Example

The "pointing mode" and"ground station visibility" state variables



The exact duration of the intervals with values **Communication**, **Visible** and **Not Visible** is uncontrollable.

The *Ground station visibility* variable is external.

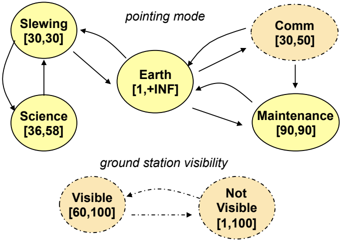
State Variables

$pm = (V, T, \gamma, D)$

- $V = \{Earth, Comm, \dots\}$
- $T(Earth) = \{Comm, Slewing, \dots\}$
  $T(Comm) = \{Earth\}, \dots$
- $\gamma(Comm) = u$,
  $\gamma(v) = c$ for the other values
- $D(Earth) = (1, \infty)$,
  $D(Slewing) = (30, 30), \dots$

$gv = (V', T', \gamma', D')$

- $V' = \{Visible, NotVisible\}$,
- $T'(Visible) = \{NotVisible\}$,
  $T'(NotVisible) = \{Visible\}$
- $\gamma(Visible) = \gamma(NotVisible) = u$
- $D'(Visible) = (60, 100)$,
  $D'(NotVisible) = (1, 100)$



Temporal Relations

- Temporal relations between intervals $A = [s_A, e_A]$ and $B = [s_B, e_B]$

$A \leq_{[lb,ub]}^{start,start} B$	A starts between lb and ub time units before B starts ($lb \leq s_B - s_A \leq ub$)
$A \leq_{[lb,ub]}^{end,end} B$	A ends between lb and ub time units before B ends ($lb \leq e_B - e_A \leq ub$)
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Allen's qualitative temporal relations can be defined

- Temporal relations between an interval $A = [s, e]$ and a timepoint t

$A \leq_{[lb,ub]}^{start/end} t$	A starts/ends between lb and ub time units before t
$A \geq_{[lb,ub]}^{start/end} t$	A starts/ends between lb and ub time units after t

Synchronization Rules

Examples

- An operational requirement: the satellite communicates with the Earth only when the ground station is visible (communication is “contained” in visibility)
- A known fact: at the beginning, the satellite is pointing to Earth

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In general

$$a_0[x_0 = v_0] \rightarrow \exists\, a_1[x_1 = v_1] \ldots a_n[x_n = v_n] \,.\mathcal{C}$$
$$\top \rightarrow \exists\, a_1[x_1 = v_1] \ldots a_n[x_n = v_n] \,.\mathcal{C}$$

Formulae: positive boolean formulae (PBFs) made up of "atoms" (temporal relations where intervals are replaced by token variables)

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Formulae: positive boolean formulae (PBFs) made up of "atoms" (temporal relations where intervals are replaced by token variables)

$$a_0[pm = Comm] \quad \rightarrow \quad \exists\, a_1[gv = Visible]\,.a_1 \leq^{start,start}_{[0,\infty]} a_0 \wedge a_0 \leq^{end,end}_{[0,\infty]} a_1$$

(a_1 contains a_0)

$$\top \quad \rightarrow \quad \exists a_1[pm = Earth].\ a_1 \leq^{start}_{[0,0]} 0$$

Planning Domains and Goals

- Planning Domain $\mathcal{D} = (SV, \mathcal{S})$
  - SV is a set of state variables
    (partitioned into planned and external state variables)
  - $\mathcal{S}$ is a set of synchronization rules
- Goal $\mathcal{G} = (\Gamma, \Delta)$
  - $\Gamma = \{g_1 = (x_1, v_1), \ldots, g_n = (x_n, v_n)\}$ (there exist intervals g_i where the variable x_i has the value v_i)
  - Δ (relational goal) is a PBF where only the token variables $g_1, \ldots g_n$ occur (the formula Δ holds for the intervals given in Γ)

  $\mathcal{G}$ is represented by the synchronization rule

$$S_{\mathcal{G}} = \top \rightarrow \exists\, g_1[x_1 = v_1] \ldots g_n[x_n = v_n].\Delta$$

Example: $\Gamma = \{g_1 = (pm, Science), g_2 = (pm, Maintenance)\}$
$\Delta = (g_1 \text{ meets } g_2) \vee (g_1 \text{ before } g_2)$

Planning Problems

- Planning Problem $(\mathcal{D}, \mathcal{G}, H)$
  - $\mathcal{D}$ is a planning domain
  - $\mathcal{G}$ is a planning goal for $\mathcal{D}$
  - $H \in \mathbb{T}$ is the planning horizon
- When external variables are present, a planning problem also contains an observation, i.e. the information available to the planner about their behavior (details in Cialdea Mayer & Orlandini & Umbrico, ACTA INFORMATICA 2016)

Non-flexible Timelines in a Controllable Context

Let $x = (V, T, D)$ be a state variable
(now ignoring the controllability tagging function)

- Token for x:

$$x^j = (v, d) \quad \text{for } j \in \mathbb{N},\ v \in V,\ d \in D(v)$$

  v is the token value: $value(x^j)$; d is its duration: $duration(x^j)$

- Timeline for x

$$TL_x = x^0 = (v_0, d_0), x^1 = (v_1, d_1), \ldots, x^k = (v_k, d_k)$$

  where $x^0, \ldots, x^k$ are tokens for x

  - e_k is the temporal horizon of TL_x.
  - once a token x^i is embedded in a timeline, its start and end times can be easily computed:

$$start\_time(x^i) = \sum_{j=0}^{i-1} d_j \qquad end\_time(x^i) = start\_time(x^i) + d_i$$

Non-flexible Plans

- A Plan for the state variables in SV is a set of timelines

$$\mathbf{TL} = \{TL_{x_1}, \ldots, TK_{x_k}\}$$

  where $SV = \{x_, \ldots, x_k\}$

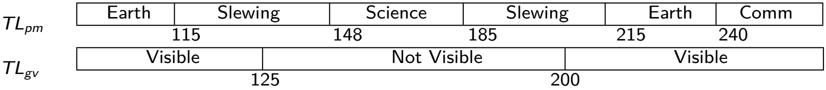
  The plan horizon H is the minimum among the temporal horizons of $TL_{x_1}, \ldots, TL_{x_k}$.

  TL describes the behavior of each state variable in SV *at least* within the time point H.

- **TL** is a solution plan for the problem $\mathcal{P} = (\mathcal{D}, \mathcal{G}, H')$ if
  - it is valid w.r.t. $\mathcal{D} = (SV, \mathcal{S})$, i.e. it satisfies all the synchronizations in $\mathcal{S}$,
  - it satisfies the synchronization rule representing $\mathcal{G}$,
  - $H \geq H'$ and
  - all the goals are fulfilled before H'

A Non-flexible Plan

- An example of non flexible plan for the satellite planning domain
- two timelines (pointing mode and ground station visibility)



Timeline-based Planning: Theory and Practice
Flexible Timelines and Dynamic Controllability

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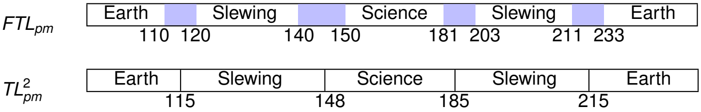
Projection of a flexible timeline: its tokens have begin and end points in the intervals of the corresponding flexible tokens.



Not every projection of a flexible timeline or plan respects the constraints of the planning domain.

Instance: a projection that is valid w.r.t. the planning domain.

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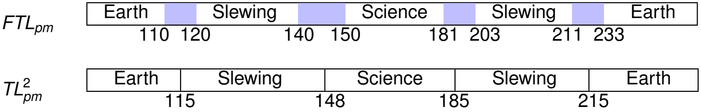


Not every projection of a flexible timeline or plan respects the constraints of the planning domain.

Instance: a projection that is valid w.r.t. the planning domain.

Goal of the formalization: describe flexible timelines and plans so that checking whether a projection is also an instance can be done without looking back at the underlying domain

The Controllability Problem

- The executor of a flexible plan must take decisions on when exactly end a given activity (token) and start the following one

  (i.e. which instance of the plan is to be executed)

- When the exact duration of some values is not under the system control, this raises controllability problems

- This part of the tutorial presents
  - a comprehensive formalization of timeline-based flexible plans
  - the definition of their controllability properties
  - a method for checking a plan dynamic controllability by exploiting existing tools for Timed Game Automata

Flexible Tokens

A flexible token for the state variable $x = (V, T, \gamma, D)$ is a tuple

$$x^j = (v, [e, e'], [d, d'], \tau)$$

for $i \in \mathbb{N}$, $v \in V$, and the obvious constraints:

$$e \leq e' \quad \text{and} \quad d_{min} \leq d \leq d' \leq d_{max} \text{ for } D(v) = (d_{min}, d_{max})$$

- x^j is the token name
- $v = value(x^j)$
- $[e, e'] = end\_time(x^j)$ is the end time interval of the token
- $[d, d'] = duration(x^j)$ is its duration interval
- $\tau = \gamma(v)$ is its controllability tag (also denoted by $\gamma(x^j)$).
    - If $\tau = c$, then x^j is a controllable token
    - if $\tau = u$, it is uncontrollable

Flexible Timelines

A (flexible) timeline FTL_x for the state variable $x = (V, T, \gamma, D)$ is a finite sequence of flexible tokens for x

$$x^0 = (v_1, [e_1, e'_1], [d_1, d'_1], \tau_1), \ldots, x^k = (v_k, [e_k, e'_k], [d_k, d'_k], \tau_k)$$

where for all $i = 1, ..., k-1$: $v_{i+1} \in T(v_i)$ and $e'_i \leq e_{i+1}$.

- $[e_k, e'_k]$ is the horizon of the timeline
- The start time interval of a token is determined by its position in a timeline:
  - $start\_time(x^0) = [0, 0]$
  - $start\_time(x^{i+1}) = end\_time(x^i)$
- A timeline for an external state variable contains only uncontrollable tokens.

Scheduled Tokens and Timelines

- A scheduled token is a token of the form

$$x^i = (v, [t, t], [d, d'], \gamma) = (v, t, [d, d'], \gamma)$$

It represents a token fixed over time ($end\_time(x^i) = t$).

A scheduled token corresponds to a non-flexible one: its end time is fixed, instead of its duration.

This new form makes scheduled tokens particular cases of flexible ones.

- A scheduled timeline TL_x is a timeline consisting of scheduled tokens only (and respecting duration constraints).

It is a schedule of a given flexible timeline if the end times of each token belong to the corresponding end time intervals.

I.e. a schedule of a flexible timeline is obtained by narrowing down to singletons (time points) the tokens end times.

- A schedule **TL** of a set of timelines **FTL** is a set of scheduled timelines where each $TL_x \in \mathbf{TL}$ is a schedule of the corresponding $FTL_x \in \mathbf{FTL}$.

Flexible Plans

A “good” plan must satisfy the synchronization rules of the domain.

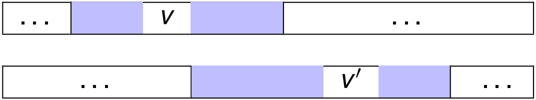
Consider, for instance

$$S = a_0[x = v] \rightarrow \exists a_1[y = v'].\, a_0 \leq^{end,start}_{[0,0]} a_1 \vee a_0 \leq^{end,start}_{[5,10]} a_1$$

and a set **FTL** of flexible timelines with tokens

x^i with $value(x^i) = v$ and $end\_time(x^i) = [30, 50]$
y^j with $value(y^j) = v'$ and $start\_time(y^j) = [30, 60]$

FTL_x =
FTL_y =



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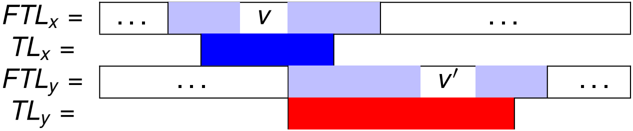

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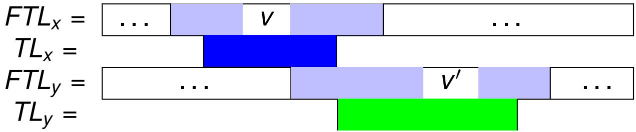

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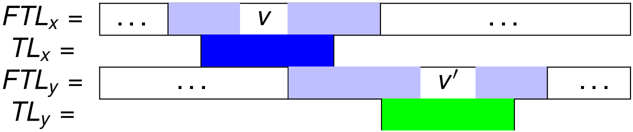

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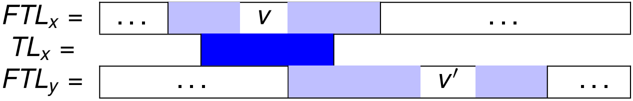


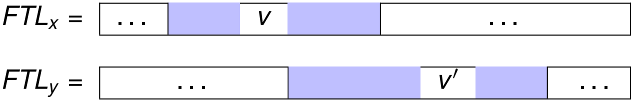
Not every pair of instances of x^i and y^j satisfies S.

The representation of a ”good” flexible plan with x^i and y^j should include the information that y^j is required to start either when x^i ends or from 5 to 10 time units after.

Flexible Plans (2)

In general, a flexible plan must include information about the relations that have to hold between tokens in order to satisfy the synchronization rules of the planning domain.

Different plans may be defined with the same set **FTL** of flexible timelines, each of them representing a possible way of satisfying the synchronization rules.



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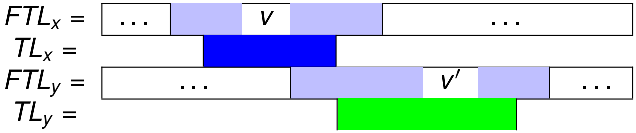

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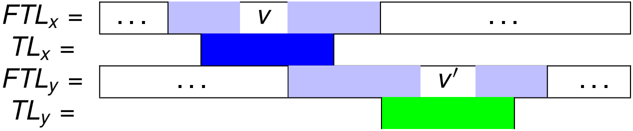

- $\Pi_2 = \textbf{FTL} + \{x^i \leq^{end,start}_{[5,10]} y^j, \ldots\}$

Flexible Plans (3)

- A flexible plan Π is a pair $(\mathbf{FTL}, \mathcal{R})$ where
  - **FTL** is a set of flexible timelines
  - $\mathcal{R}$ is a set of relations on tokens in **FTL**.
- An instance of the flexible plan $\Pi = (\mathbf{FTL}, \mathcal{R})$ is any schedule of **FTL** satisfying every relation in $\mathcal{R}$.

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- A flexible plan represents the set of its instances.
- $\mathcal{R}$ enforces the plan to obey the rules of planning domains and to achieve the goals
- The pair $(\mathbf{FTL}, \mathcal{R})$ describes all the information required to execute the plan

Semantics of Synchronization Rules on Flexible Plans

A plan $\Pi = (\textbf{FTL}, \mathcal{R})$ satisfies a synchronization rule S if:

the relations in $\mathcal{R}$ hold $\Longrightarrow$ the constraints represented by S hold

In other terms, $\mathcal{R}$ represents a possible choice to satisfy S.

Example

Consider the rule S:

$$a_0[pm = Comm] \rightarrow \quad \exists\, a_1[gv = Visible]\,.a_1 \leq^{start,start}_{[0,\infty]} a_0 \wedge a_0 \leq^{end,end}_{[0,\infty]} a_1$$

(i.e. a_1 contains a_0)

and timelines:

FTL_{pm} with $pm^5 = (Comm, [80, 120], [30, 50], u)$, with start time $[50, 70]$

FTL_{gv} with $gv^4 = (Visible, [120, 190], [60, 100], u)$ with start time $[60, 90]$

The flexible plan

$$\Pi = (\mathbf{FTL}, \mathcal{R})$$

with

$$\mathbf{FTL} = \{FTL_{pm}, FTL_{gv}\} \text{ and}$$

$$\mathcal{R} = \{gv^4 \leq^{start,start}_{[0,\infty]} pm^5, pm^5 \leq^{end,end}_{[0,\infty]} gv^4\}$$

satisfies S, because mapping a_0 to pm^5 and a_1 to gv^4 makes a_1 contains a_0 true.

Valid Flexible Plans

A flexible plan $\Pi = (\textbf{FTL}, \mathcal{R})$ is valid w.r.t. a planning domain $\mathcal{D} = (SV, \mathcal{S})$ if:

- it is complete: Π satisfies all the synchronization rules in $\mathcal{S}$;
- it is consistent: it has at least an instance.

Π is a flexible solution plan for $\mathcal{P} = (\mathcal{D}, \mathcal{G}, H)$ if

- it is valid w.r.t. $\mathcal{D}$,
- it satisfies the synchronization rule representing $\mathcal{G}$,
- the horizon of every timeline for a planned state variable is $[H, H]$

Theorem. If the flexible plan Π is complete w.r.t. the planning domain $\mathcal{D}$, then every instance of Π is valid w.r.t. $\mathcal{D}$.

Consequence: if Π is valid w.r.t. $\mathcal{D}$ then there exists an instance of Π that is valid w.r.t. $\mathcal{D}$.

Controllability: Flexible Plans and STNU

- A formal equivalence between STNU and flexible plans is missing

  [Morris, Muscettola, Vidal 2001, Cesta et al 2009]

- Taking inspiration from the work on STNU, the same concepts can be defined for flexible plans
- Given a plan $\Pi = (\mathbf{FTL}, \mathcal{R})$, we consider

$$\textit{tokens}(\mathbf{FTL}) = \textit{tokens}_C(\mathbf{FTL}) \cup \textit{tokens}_U(\mathbf{FTL})$$

- Duration constraints and temporal relations on $\textit{tokens}_U$ correspond to contingent links

Situations and Projections

- Given a set of timelines **FTL**, a situation ω is a total function

$$\omega : \textit{tokens}_U(\mathbf{FTL}) \rightarrow \mathbb{T}$$

  where $\omega(x^i)$ is in $\textit{duration}(x^i)$.

  A situation is a function assigning a (legal) value to the duration of each uncontrollable token.

- The set of all situations for **FTL** is denoted by $\Omega_{\mathbf{FTL}}$
- A situation ω for **FTL** defines a projection $\omega(\mathbf{FTL})$ of **FTL** – i.e. a fully controllable evolution of **FTL**:

  every uncontrollable token $x^i = (v, [e, e'], [d, d'])$ in **FTL** is replaced, in $\omega(\mathbf{FTL})$, by $(v, [e, e'], \omega(x^i))$.

Scheduling and Execution Strategy

- A scheduling function θ assigns an execution time to the end time of each token

$$\theta : \textit{tokens}(\textbf{FTL}) \rightarrow \mathbb{T}$$

- The set of all the scheduling functions is denoted by $T_{\textbf{FTL}}$
- A scheduling function θ for a flexible plan $(\textbf{FTL}, \mathcal{R})$ is consistent iff the scheduled timelines induced by θ are an instance of the plan
- An execution strategy for a flexible plan is a mapping

$$\sigma : \Omega_{\textbf{FTL}} \rightarrow T_{\textbf{FTL}}$$

It is viable if for each situation ω the scheduling function $\sigma(\omega)$ is consistent with the plan $(\omega(\textbf{FTL}), \mathcal{R})$

Prehistory and DES

- If $t \in \mathbb{T}$, the prehistory $\theta_{\prec t}$ is a partial function defined only for uncontrollable tokens

$$\theta_{\prec t} : \mathit{tokens}_U(\mathbf{FTL}) \to \mathbb{T}$$

  It assigns a duration to uncontrollable tokens that finish before t according to θ.

- A prehistory defines a partial situation, i.e. a partial projection of **FTL**
- A dynamic execution strategy for a plan is an execution strategy σ for **FTL** such that for all situations ω, ω' and every controllable token x^i:

  if $\sigma(\omega) = \theta$,
  $\sigma(\omega') = \theta'$
  and $\theta(x^i) = t$,
  then $\theta_{\prec t} = \theta'_{\prec t}$ implies $\theta(x^i) = \theta'(x^i)$

Controllability of Flexible Plans

A Flexible Plan $\Pi = (\mathbf{FTL}, \mathcal{R})$ is

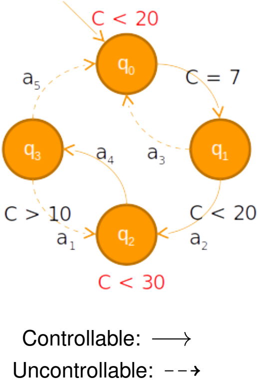
- Weakly controllable if there is a viable execution strategy for Π
- Strongly controllable if there is a viable execution strategy for Π giving the same scheduling function for every situation
- Dynamically controllable if there is a *dynamic execution strategy* (DES) for Π – decisions only consider past uncontrollable events

Dynamic controllability constitutes a highly desirable property for a flexible plan

Timed Game Automata

[Maler & Pnueli & Sifakis 1995]

- The set *Act* of *actions* is split in two disjoint sets
  - Act_c: the set of controllable actions
  - Act_u: the set of uncontrollable actions
- A valuation is a mapping from the set of clocks to integers
- A state is a pair (q_i, v) with v a valuation
- A strategy F is a partial mapping from the set of Runs of A to the set $Act_c \cup \{\lambda\}$
- The special action λ stands for "just wait and do nothing"



Building TGA from Timelines

- A Flexible Plan $(\mathbf{FTL}, \mathcal{R})$ is encoded into a network of TGA
  - Each TL_x in **FTL** is encoded by an automaton, a location for each token
  - Transition controllability is defined according to tokens controllability tags
  - Temporal relations are encoded by clock constraints on transitions
- A TGA Reachability Game (RG) is defined so that
  - Winning the game implies checking DC for a flexible plan
- UPPAAL-TIGA is used as verification engine
  - The winning strategy is a viable DES for the encoded plan

The encoding tool plan2tiga and details are available at
http://cialdea.dia.uniroma3.it/plan2tiga

Empirical Evaluation

- Aim: investigating the practical feasibility of the TGA- based approach
- Approach:
  - APSI-TRF and EPSL as the planning engine
  - A benchmark domain inspired by a Space Long Term Mission Planning problem
- Results: the experiments show the feasibility of the approach in realistic scenarios
- Details in M. Cialdea Mayer & A. Orlandini, TIME 2015.