FIRST-ORDER AUTOMATA

Nicola Gigante
Free University of Bozen-Bolzano, Italy

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Infinite-state specification and verification

Model-checking and other verification techniques for **finite-state** systems have been extremely successful in the past decades.

However, many real-world scenarios require reasoning over **infinite-state** systems.

Infinite-state specification and verification

There are **many** infinite-state formalisms around.

- timed automata [AD94]
- hybrid automata [Alu+92]
- recursive state machines [Alu+05; BMP10]
- visibly pushdown languages [AM04]
- operator-precedence languages [Dro+17]
- FIFO machines [BZ83]
- counter machines [Woj99]
- Petri nets [Mur89; JK09]
- data-aware systems [DLV19; CGM13; Ghi+23]
- automata over infinite alphabets [Seg06; IX19]
- register automata [KF94]

Infinite-state specification and verification

And many **logics** to talk about them.

- precedence oriented temporal logic [CMP22]
- metric temporal logics [Koy90; LVR22; AH94]
- temporal logics with concrete domains [DQ21]
- temporal logics with arithmetics [Fel+23; Cim+20]
- separation logic [CYO01; BK18; Man20]
- constrained horn clauses [GB19]
- logics on data words [Boj+06; DLN07]
- signal temporal logic [MN04]
- first-order temporal logics [Kon+04]

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- signal temporal logic [MN04]
- **first-order temporal logics** [Kon+04]

First-order Temporal Logic

First-order temporal logics are expressive formalisms to specify properties of infinite-state systems:

- badly undecidable, however;
- used mostly in really tailored fragments or in **knowledge representation** applications where reasoning is not the main task;

Question

Can we make them **practical**, as a specification language for **verification** tasks, while maintaining a substantial fraction of its **expressive power**?

First-order automata

In the finite-state setting, **automata** have played the crucial role of the algorithmic powerhorse of verification and model checking.

Question

Can we define a similar tool for **first-order** temporal logic?

Table of contents

High-level overview of the talk:

- recap on propositional and first-order temporal logics
- past results: LTL modulo theories
- current directions: first-order automata
- conclusions

FIRST-ORDER TEMPORAL LOGICS

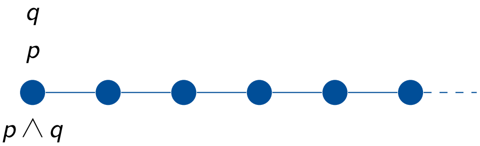
Linear Temporal Logic

Let us recap the main concepts of Linear Temporal Logic:

- modal logic interpreted over **discrete linear orders**
- traditionally, **infinite** linear orders
- recently, **finite** models gained attention [DV13]

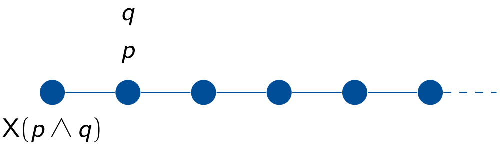
Linear Temporal Logic

Linear Temporal Logic

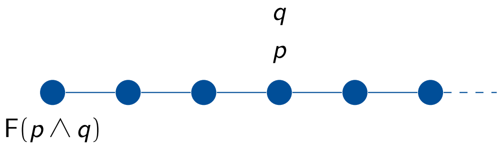


p and q hold

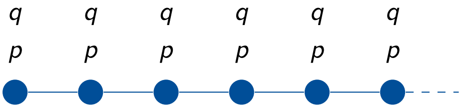
Linear Temporal Logic



tomorrow p and q hold



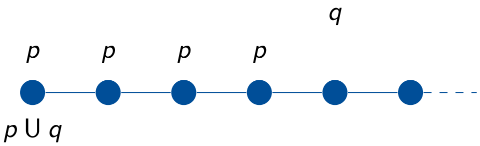
eventually p and q hold



$\mathsf{G}(p \wedge q)$

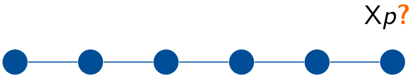
p and q hold **always**

Linear Temporal Logic

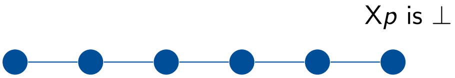


p holds **until** q holds

Linear Temporal Logic over Finite Traces


Xp?

Linear Temporal Logic over Finite Traces



Linear Temporal Logic over Finite Traces

$\widetilde{\mathsf{X}}p \equiv \neg\mathsf{X}\neg p$ is $\top$

weak tomorrow

Linear Temporal Logic over Finite Traces

$\widetilde{\mathsf{X}}\bot$ is $\top$

$\mathsf{X}\top$ is $\bot$

First-order temporal logic

In FOLTL we mix classical first-order logic with LTL temporal operators.

$$\forall x \exists y \, \mathsf{G}\big(p(x) \rightarrow \mathsf{F}(q(x, y))\big) \qquad \mathsf{G}\big(\forall xy \, (p(x, y) \rightarrow \widetilde{\mathsf{X}} p(x, y + 1))\big)$$

$$\forall x \, \mathsf{X} \neg p(x) \land \mathsf{X} \exists x \, p(x)$$

First-order temporal logic

FOLTL is interpreted over **sequences of first-order structures**.

- possibly over **multisorted** signatures;
- first-order features interpreted over the current structure;
- temporal operators to move from one structure to another;
- the **domain** of each sort is arbitrary but **constant** throughout the sequence.

First-order temporal logic

FOLTL is famous for being **highly undecidable**, but, for a fragment F such that:

- the underlying FO fragment is decidable; and
- temporal formulas are **monodic**;
  - only one **free variable** inside temporal operators;
- constants in the signature are considered **rigid**;
- then FOLTL(F) is decidable [HWZ00]

Decidable fragments

Are there decidable fragments of FO?

- monadic fragment
- Bernays-Schönfinkel-Ramsey fragment ($\exists^*\forall^* / =$)
- Ackermann fragment ($\exists^*\forall\exists^*$)
- Gödel-Kalmár-Schütte fragment ($\exists^*\forall\forall\exists^*$)
- Skolem fragment
- two-variable fragment
- guarded fragment
- separated fragment
- combinations of the above in multi-sorted signatures
- many description logics are decidable FO fragments in disguise

See [Voi19] for a detailed survey.

THE PAST:
LTL MODULO THEORIES

FOLTL in practice

There have been many developments in the world of first-order theorem proving.

- many good solvers (SPASS, Vampire, E, *etc.*);
- not specific to decidable fragments, however.

Can we use these as a foundation of FOLTL reasoning? Yes.

- See *e.g.*, TSPASS, a solver for monodic first-order temporal logic

FOLTL in practice

However, the **monodic** temporal fragment is limited for applications.

Imagine modeling an **update** query on an SQL database:

update R **set** x = x - 1 **where** x > 0

In FOLTL, it can become something like:

$$\forall k \forall x \left(\begin{array}{l} \quad \big(R(k,x) \wedge x > 0 \rightarrow \mathsf{X}(\neg R(k,x) \wedge R(k,x-1))\big) \\ \wedge \big(R(k,x) \wedge x \leqslant 0 \rightarrow \mathsf{X} R(k,x)\big) \end{array} \right)$$

We need to accept dealing with **semi-decidability**.

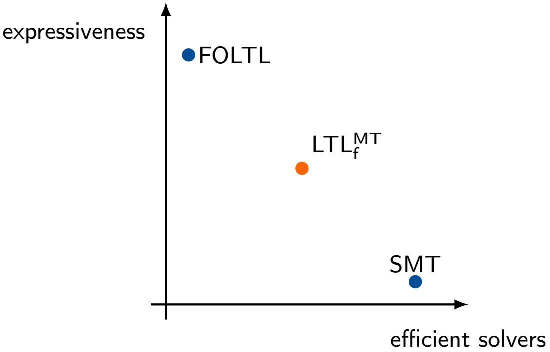
Satisfiability Modulo Theories

There is another approach at **first-order** reasoning:

- satisfiability modulo theories
- **bottom-up** approach:
  - let's add to SAT solvers bits of tractable first-order reasoning step by step
- SMT solvers are strong at quantifier-free reasoning over specific theories useful for program verification
  - linear integer arithmetics/real arithmetics
  - arrays
  - bitvectors
  - constrained Horn clauses
  - algebraic data types
- sometimes quantification works, as well

LTL_f modulo theories

LTL_f^{MT} is a sweet spot in the specification of infinite-state properties. [IJCAI 22]



$\mathsf{LTL_f}$ modulo theories

LTL_f^{MT} extends $\mathsf{LTL_f}$ by introducing **some** first-order elements.

- let Σ be a first-order signature and T a Σ-theory
- LTL_f^{MT} is interpreted over finite words of T-structures
- predicates and function symbols are **rigid**
- constants are **non-rigid**, *i.e.*, change over time
  - this is **not** covered by existing decidability results about FOLTL

$\mathsf{LTL_f}$ modulo theories

$\mathsf{LTL_f^{MT}}$ extends $\mathsf{LTL_f}$ by introducing **some** first-order elements.

- terms can refer to the value of constants at the **next** or **previous** state.

$$t \coloneqq c \mid x \mid f(t_1, \ldots, t_n) \mid \bigcirc c \mid \circledsim c \mid \lhd c \mid \tilde{\lhd} c$$

- $\mathsf{LTL_f}$ propositions are replaced by first-order sentences

$$\lambda \coloneqq p(t_1, \ldots, t_n) \mid \neg\lambda \mid \lambda \vee \lambda \mid \lambda \wedge \lambda \mid \exists x\, \lambda \mid \forall x\, \lambda$$

$$\phi \coloneqq \lambda \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \mathsf{X}\phi \mid \widetilde{\mathsf{X}}\phi \mid \mathsf{Y}\phi \mid \mathsf{Z}\phi$$
$$\mid \phi\, \mathsf{U}\, \phi \mid \phi\, \mathsf{R}\, \phi \mid \phi\, \mathsf{S}\, \phi \mid \phi\, \mathsf{T}\, \phi$$

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$$\begin{aligned}
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\phi &\coloneqq \lambda \mid \neg\phi \mid \phi \lor \phi \mid \phi \land \phi \mid \mathsf{X}\phi \mid \widetilde{\mathsf{X}}\phi \mid \mathsf{Y}\phi \mid \mathsf{Z}\phi \\
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\end{aligned}$$

Examples

$$\mathsf{G}(a = 2b) \qquad (a < b)\,\mathsf{U}\,(b = 0) \qquad \mathsf{G}(a > 5) \wedge \mathsf{F}(a = 0)$$

$$\mathsf{G}(\exists x (a = 2x))$$

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Examples

$$a = 0 \wedge ((\bigcirc\, a = a + 1)\ \mathsf{U}\ a = 42)$$

$$b = 1 \wedge \mathsf{G}(\circledtilde\, b = b + 1 \wedge a = 2b)$$

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(Un)decidability

$\mathsf{LTL}_f^{\mathsf{MT}}$ is **undecidable**, but:

- it is **semi-decidable**
- so we can get something useful, sometimes:
  - models of satisfiable formulas
  - counterexamples of specifications over bugged systems
- moreover, we have some interesting **decidable** fragments [ECAI 23]
- why **finite traces**?
  - on **infinite traces**, $\mathsf{LTL}^{\mathsf{MT}}$ is not even semi-decidable.

Decidable fragments

(MC) Formulas over LRA where all **iteration conditions** are **monotonicity constraints**, *i.e.*, variable-to-variable or variable-to-constant comparisons, *e.g.*:

$$a < 0 \wedge b = 1 \wedge ((\bigcirc b > b \wedge \bigcirc a \leqslant a) \,\mathsf{U}\, a = b)$$

(IPC) Formulas over LIA where all **iteration conditions** are **integer periodicity constraints**, *e.g.*:

$$(b \equiv_3 a) \,\mathsf{U}\, (a > 42) \wedge \mathsf{F}(a + b = c)$$

Iteration conditions:

- α in $\alpha \,\mathsf{U}\, \beta$
- β in $\alpha \,\mathsf{R}\, \beta$

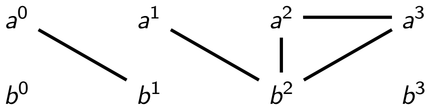
Decidable fragments

(BL) **Bounded lookback** formulas, that generalize the above two by requiring that cross-state interaction is restricted to finitely many steps back. What does it mean?

Decidable fragments

Bounded lookback formula

$$p(a, \bigcirc b) \,\mathsf{U}\, (\bigcirc a = a + b)$$



$p(a^0, b^1)$ $\quad$ $p(a^1, b^2)$ $\quad$ $a^3 = a^2 + b^2$

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Decidable fragments

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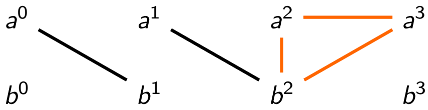

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Decidable fragments

Non-BL formula

$$a < 0 \wedge b = 1 \wedge ((\bigcirc\, b > b \wedge \bigcirc\, a \leqslant a) \,\mathsf{U}\, a = b)$$

$$\begin{tikzcd}
a^0 \arrow[r, dash] & a^1 \arrow[r, dash] & a^2 \arrow[r, dash] & a^3 \arrow[d, dash] \\
b^0 \arrow[r, dash] & b^1 \arrow[r, dash] & b^2 \arrow[r, dash] & b^3
\end{tikzcd}$$

$$\begin{array}{cccc}
a^1 \leqslant a^0 & a^2 \leqslant a^1 & a^3 \leqslant a^2 & a^3 = b^3 \\
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Decidable fragments

(NCS) Formulae without **cross-state comparisons**, *i.e.*, without any $\bigcirc c$, $\circledsim c$, *e.g.*:

$$(a > b \,\mathsf{U}\, a + b = 2c) \wedge \mathsf{G}(a + b > 0)$$

(FX) Formulas where the only temporal operators are F, X, and $\widetilde{\mathsf{X}}$, *e.g.*:

$$\mathsf{F}(p(\bigcirc a) \wedge \mathsf{X}(\neg p(a))) \wedge \mathsf{XF}(r(a, b) \vee r(\bigcirc a, b))$$

The SAT encoding of Reynolds' tableau can be extended to an **SMT encoding** for $\mathrm{LTL}_\mathrm{f}^\mathrm{MT}$.

- implemented in our BLACK solver[1] for arithmetic theories;
- given to **off-the-shelf** SMT solvers such as Z3 [MB08], cvc5 [Bar+22], *etc.*

[1] `https://www.black-sat.org`

Verification of $\mathrm{LTL}_f^{\mathrm{MT}}$ properties

Which systems can we verify $\mathrm{LTL}_f^{\mathrm{MT}}$ formulas on?

Knowledge-base driven Dynamic Systems (KDS):

- infinite-state transition systems

$$D = \langle \mathsf{K}, I(X), T(X, X'), F(X) \rangle$$

- states are structures over the **first-order theory** K
- $I(X)$, $T(X, X')$, $F(X)$ are arbitrary **first-order** formulas over the theory K
  - **initial** states satisfying $I(X)$
  - **final** states satisfying $F(X)$
  - **transition relation** expressed by $T(X, X')$

Verification of KDSs

Let D be a KDS and ϕ an $\mathrm{LTL}_f^{\mathrm{MT}}$ formula:

- all the executions of a KDS D can be **represented** by an $\mathrm{LTL}_f^{\mathrm{MT}}$ formula ψ_D
- **model-checking** of ϕ over D reduces to **satisfiability** of:

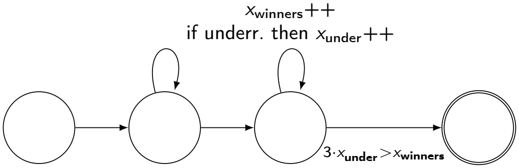
$$\gamma \equiv \psi_D \wedge \neg\phi$$

- if γ is **satisfiable**, the specification does not hold and the model is a counterexample
- if γ is **unsatisfiable**, the specification is valid over D
- this needs **non-rigid predicates**: only semi-decidable!

Some early experiments

Test setting:

- simulation of a company hiring process
- nondeterministic transitions:
  - dependent on arithmetic constraints
  - acting on unbounded relational data
- minimal length of the counterexamples dependent over scalable parameter N
- two modelings of the same system:
  - P_1 employs arithmetic constraints
  - P_2 avoids arithmetics, simulates constraints by other means
- two different properties for each variant



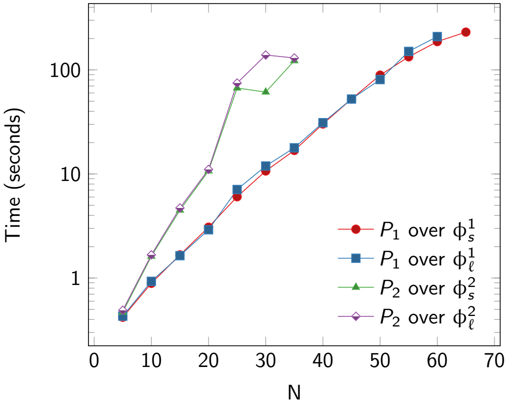
$$\phi_s^1 \equiv \mathsf{G}(x_{state} = final \rightarrow 2x_{\text{under}} > x_{\text{winners}})$$

$$\phi_\ell^1 \equiv \mathsf{G}\begin{pmatrix} x_{state} = app \rightarrow \\ F(x_{\text{state}} = final \wedge 2x_{\text{under}} > x_{\text{winners}}) \end{pmatrix}$$

Some early experiments

Results:

- 5 minutes timeout reached at $N = 70$
- **exponential** growth
  - but could be much worse,
    the problem is **undecidable**!
- liveness property not harder than the safety one
- system with **explicit arithmetics** faster to verify



THE PRESENT:
FIRST-ORDER AUTOMATA

Going beyond $\mathsf{LTL}_f^{\mathsf{MT}}$

$\mathsf{LTL}_f^{\mathsf{MT}}$ is limited in many ways:

- many complex systems need **evolving predicates**;
- **nesting** of quantifiers and temporal operators is often essential
- we want to approach **full** FOLTL.

The need for automata

In the finite-state setting, automata are essential:

- operational counterpart of temporal logics
- basis of many algorithms and techniques

We want a class of automata for FOLTL.

First-order automata

What may an infinite-state first-order automaton look like?

- **states**: first-order structures over a state signature Γ;
- **letters**: first-order structures over an alphabet signature Σ;
- **initial states**: a class of Γ-structures;
- **final states**: a class of Γ-structures;
- **transition**: a relation between:
  - a Γ-structure (source state),
  - a Σ-structure (letter), and
  - another Γ-structure (dest. state).

But this is algorithmically untractable.

Symbolic representation

A **finite-state** automaton:

$$A = \langle \Sigma, Q, Q_0, \Delta, F \rangle$$

A **symbolic** finite-state automaton:

$$\mathcal{A} = \langle \Sigma, X, I(X), T(X, \Sigma, X'), F(X) \rangle$$

Symbolic representation

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A **symbolic** finite-state automaton:

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A **first-order** automaton:

$$\mathrm{A} = \langle \Sigma, \Gamma, \phi_I, \phi_T, \phi_F \rangle$$

where:

- Σ is the **word** signature and Γ is the **state** signature;
- ϕ_I and ϕ_F are Γ-sentences;
- ϕ_T is a sentence over $\Gamma \cup \Sigma \cup \Gamma'$.

First-order automata

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A word (sequence of Σ-structures) $\overline{\sigma} = \langle \sigma_0, \dots, \sigma_{n-1} \rangle$ is accepted iff there is a **trace** (sequence of Γ-structures) $\overline{\rho} = \langle \rho_0, \dots, \rho_n \rangle$ such that:

- $\rho_0 \models \phi_0$;
- $\rho_i \cup \sigma_i \cup \rho'_{i+1} \models \phi_T$;
- $\rho_n \models \phi_F$.

First-order T-regular languages

A set of first-order words whose structures come from a theory T is a **T-language**.

A T-language is **first-order T-regular** if there is a first-order automaton accepting it.

First-order T-regular languages

Theorem (Closure properties)

*First-order T-regular languages are **closed** by:*

- *union;*
- *intersection;*
- *concatenation;*
- *Kleene star.*

As an example we see how to prove concatenation.

Concatenation

We have two automata $A_1 = \langle \Sigma, \Gamma_1, \phi_0^1, \phi_T^1, \phi_F^1 \rangle$ and $A_2 = \langle \Sigma, \Gamma_2, \phi_0^2, \phi_T^2, \phi_F^2 \rangle$.

We want $A = \langle \Sigma, \Gamma, \phi_0, \phi_T, \phi_F \rangle$ such that $\mathcal{L}(A) = \mathcal{L}(A_1) \cdot \mathcal{L}(A_2)$.

- suppose for a moment that ϕ_T can be an **existential second-order** sentence;
- then, what about:

$$\begin{aligned} \phi_T \equiv (p \wedge \phi_T^1 \wedge p') \vee (\neg p \wedge \phi_T^2 \wedge \neg p') \vee \\ \Big(p \wedge \neg p' \wedge \phi_F^1 \wedge \exists \Gamma'' \big(\phi_0^2[\Gamma/\Gamma''] \wedge \phi_T^2[\Gamma/\Gamma'']\big)\Big) \end{aligned}$$

with $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \{p\}$.

Concatenation

What about these existential second-order quantifiers?

$$\phi_T \equiv (p \wedge \phi_T^1 \wedge p') \vee (\neg p \wedge \phi_T^2 \wedge \neg p') \vee \\ \Big(p \wedge \neg p' \wedge \phi_F^1 \wedge \exists \Gamma'' \big(\phi_0^2[\Gamma/\Gamma''] \wedge \phi_T^2[\Gamma/\Gamma'']\big)\Big)$$

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Concatenation

What about these existential second-order quantifiers?

$$\phi_T \equiv \exists \Gamma'' \begin{bmatrix} (p \wedge \phi^1_T \wedge p') \vee (\neg p \wedge \phi^2_T \wedge \neg p') \vee \\ \left(p \wedge \neg p' \wedge \phi^1_F \wedge (\phi^2_0[\Gamma/\Gamma''] \wedge \phi^2_T[\Gamma/\Gamma''])\right) \end{bmatrix}$$

Now the predicates in Γ'' can be added to Γ itself:

- the semantics of the automata will require the existence of their interpretation;
- we get back a first-order automaton on an extended state signature.

Concatenation

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- the semantics of the automata will require the existence of their interpretation;
- **we get back a first-order automaton on an extended state signature.**

Complementation

Complementation is missing from the picture:

- easy for **deterministic** automata;
- so **determinization** is the key;
- how to symbolically represent the subset construction of a first-order automaton?
  - we are working on it. . .

Capturing FOLTL

A major feature of automata is that they capture temporal logic.

Can we capture FOLTL with first-order automata?

Capturing FOLTL

We need some ingredients.

Definition (Stepped normal form)

Given an FOLTL formula ϕ, the *step normal form* of ϕ, denoted $\mathsf{snf}(\phi)$, is the formula defined recursively as follows:

- $\mathsf{snf}(p(t_1, \ldots, t_n)) = p(t_1, \ldots, t_n)$ and $\mathsf{snf}(t_1 = t_2) = (t_1 = t_2)$;
- $\mathsf{snf}(Qx\phi) = Qx\,\mathsf{snf}(\phi)$, where $Q \in \{\forall, \exists\}$ and x is a first-order variable;
- $\mathsf{snf}(\neg\phi) = \neg\,\mathsf{snf}(\phi)$;
- $\mathsf{snf}(\phi_1 \circ \phi_2) = \mathsf{snf}(\phi_1) \circ \mathsf{snf}(\phi_2)$, where $\circ \in \{\wedge, \vee\}$
- $\mathsf{snf}(\circ\phi) = \circ\phi$ where $\circ \in \{\mathsf{X}, \mathsf{Y}, \widetilde{\mathsf{X}}, \mathsf{Z}\}$;
- $\mathsf{snf}(\phi_1 \mathbin{\mathsf{U}} \phi_2) = \mathsf{snf}(\phi_2) \vee (\mathsf{snf}(\phi_1) \wedge \mathsf{X}(\phi_1 \mathbin{\mathsf{U}} \phi_2))$;
- $\mathsf{snf}(\phi_1 \mathbin{\mathsf{R}} \phi_2) = \mathsf{snf}(\phi_2) \wedge (\mathsf{snf}(\phi_1) \vee \widetilde{\mathsf{X}}(\phi_1 \mathbin{\mathsf{R}} \phi_2))$.

Definition (Closure)

The *closure* of a FOLTL sentence ϕ is the set $C(\phi)$ defined as follows:

- $X\phi \in C(\phi)$;
- $\psi \in C(\phi)$ for any subformula ψ of ϕ (including itself);
- for any $\phi_1 \mathbin{U} \phi_2 \in C(\phi)$ we have $X(\phi_1 \mathbin{U} \phi_2) \in C(\phi)$;
- for any $\phi_1 \mathbin{R} \phi_2 \in C(\phi)$ we have $\widetilde{X}(\phi_1 \mathbin{R} \phi_2) \in C(\phi)$.

Capturing FOLTL

We also define the following subsets of the closure:

$$\mathsf{XS} = \{\mathsf{xs}_\psi \mid \mathsf{X}\psi \in \mathsf{C}(\phi)\}$$
$$\widetilde{\mathsf{X}}\mathsf{S} = \{\mathsf{ws}_\psi \mid \widetilde{\mathsf{X}}\psi \in \mathsf{C}(\phi)\}$$

where xs_ψ and ws_ψ are predicates of the arity n corresponding to the number of free first-order variables in ψ.

Capturing FOLTL

Let ϕ be a FOLTL sentence. The automaton of ϕ is $\mathcal{A}(\phi) = \langle \Sigma, \Gamma, \phi_0, \phi_T, \phi_F \rangle$ where:

- the state signature is $\Gamma = \mathsf{XS} \cup \widetilde{\mathsf{X}}\mathsf{S}$;
- the initial and final conditions are:

$$\phi_0 \equiv \mathsf{xs}_\phi$$

$$\phi_F \equiv \bigwedge_{\mathsf{ws}_\psi \in \widetilde{\mathsf{X}}\mathsf{S}} \forall \overline{x}.\mathsf{ws}_\psi(\overline{x}) \wedge \bigwedge_{\mathsf{xs}_\psi \in \mathsf{XS}} \forall \overline{x}.\neg \mathsf{xs}_\psi(\overline{x})$$

- the transition relation is:

$$\phi_T \equiv \bigwedge_{s_\psi \in \mathsf{XS} \cup \widetilde{\mathsf{X}}\mathsf{S}} \forall \overline{x}.[s_\psi(\overline{x}) \leftrightarrow \mathsf{snf}'_S(\psi(\overline{x}))]$$

Checking (non-)emptiness

Of course checking emptiness of first-order automata is **undecidable**.

However, we can state a semi-algorithm for non-emptiness with some assumptions.

Checking (non-)emptiness

For $k > 0$, let:

$$[\![A]\!]_k^F \equiv \phi_0^0 \wedge \bigwedge_{i=0}^{k-1} \phi_T^k \wedge \phi_F^k$$

```
procedure NonEmpty(A)
    k ← 0
    while true do
        if [[A]]_k^F is satisfiable then
            return not empty
        end if
        k ← k + 1
    end while
end procedure
```

Decidable fragments

Recall the result from the literature we cited before.

For a fragment F such that:

- the underlying FO fragment is decidable; and
- temporal formulas are **monodic**;
  - only one **free variable** inside temporal operators;
- constants in the signature are considered **rigid**;
- then FOLTL(F) is decidable [HWZ00]

Decidable fragments

Can we recover this result from first-order automata? **Yes.**

- if we follow the encoding we notice that **monodic** FOLTL sentences translate into automata with **monadic** state signature Γ;
- monadic predicates are easier to deal with (*e.g.*, monadic FO is decidable);
- we can translate a monadic Γ into a Γ containing only **propositions**;
- emptyness for this kind of automata is decidable (they are almost finite-state).

CONCLUSIONS

First-order automata in practice

Our satisfiability checking tool BLACK is currently being refactored to support full FOLTL through first-order automata.

- experimental results soon, maybe another iFM^2 talk?

Better algorithms

Can we go beyond the simple unraveling semi-algorithm shown before?

For better **practical** results we need more **theory**.

Interesting paths we are exploring:

- fixpoint computation based on **second-order quantifier elimination** [GSS08];
- encoding into **constrained Horn clauses** [GB19];

What we have not seen today

We can add **past operators** to FOLTL.

- if we start from **pure-past** FOLTL sentences, we directly obtain **deterministic** first-order automata;
- this happens with pure-past LTL as well and is crucial for **reactive synthesis**;
- 2EXPTIME for LTL, EXPTIME for pure-past LTL, because of this fact;
- can this help with **reactive synthesis** for FOLTL specifications?

Conclusions

We want first-order temporal logics to become practical tools for infinite-state specification and verification.

- long way ahead;
- little steps already done;
- first-order automata as a reasoning and algorithmic tool;
- many theoretical and algorithmic developments missing yet.

Conclusions

We want first-order temporal logics to become practical tools for infinite-state specification and verification.

- long way ahead;
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- many theoretical and algorithmic developments missing yet.

Thank you!

Questions?

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