Incremental Inductive Verification

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Introduction

Program verification - Timeline

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$$\phi P \psi$$

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1986. Model checking (fully automatic) [CES86]:

$$\mathcal{M}, \sigma \models \phi \qquad \forall \text{ paths } \sigma$$

where $\mathcal{M}$ is a representation of machine and ϕ is a temporal formula.

(Symbolic) Transition System

- (Symbolic) Finite-state transition system $\mathcal{M} = (\bar{i}, \bar{x}, I, T)$
  - $\bar{i}$ is a set of input variables;
  - $\bar{x}$ is a set of state variables;
  - $I(\bar{x})$ is the formula for initial states;
  - $T(\bar{x}, \bar{i}, \bar{x}')$ is the formula for the transition relation;

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$$\begin{aligned} s &= x_1 \land \neg x_2 \land x_3 \land \neg x_4 \land \neg x_5 \\ &= \langle 1, 0, 1, 0, 0 \rangle \end{aligned}$$

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- a trace of the system is a sequence $s_0, s_1, \ldots$ such that $s_0 \models I$ and $s_i, s'_{i+1} \models T \; \forall i \geq 0$.

(Symbolic) Transition System

- a boolean formula F over $\bar{x}$ denotes the set of states $[\![F]\!] = \{s \in \{0,1\}^n \mid s \models F\}$:

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- a clause c is a disjunction of literals. A subclause $d \subseteq c$ is a clause whose literals are a subset of c's literals. It holds that:

$$d \Rightarrow c$$

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- Invariant property $P(\bar{x})$: boolean formula that asserts that only P-states are reachable.
- P is $\mathcal{M}$-invariant if $P(\bar{x})$ holds for system $\mathcal{M}$. If this is not the case, there exists a counterexample trace $s_0, s_1, \ldots, s_k$ such that $s_k \not\models P$.

$$\Rightarrow \text{reachability problem}$$

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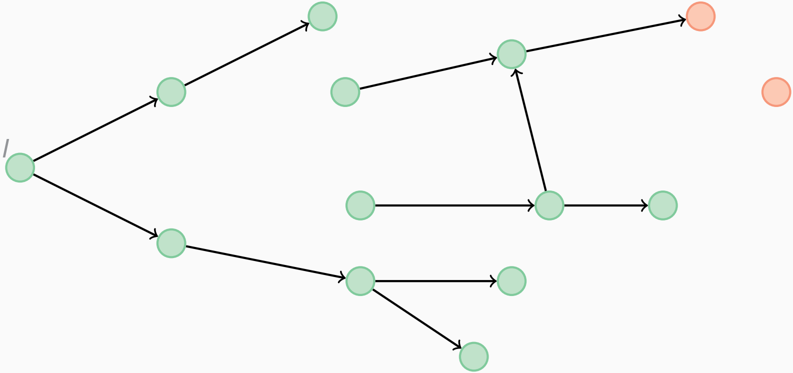
2. BDD-based techniques [McM93]: state-space explosion problem;
3. Bounded Model Checking (BMC) [Bie+99]: unrolling of the transition relation;

Symbolic Algorithms for Reachability

BDD-based backward algorithms

Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.



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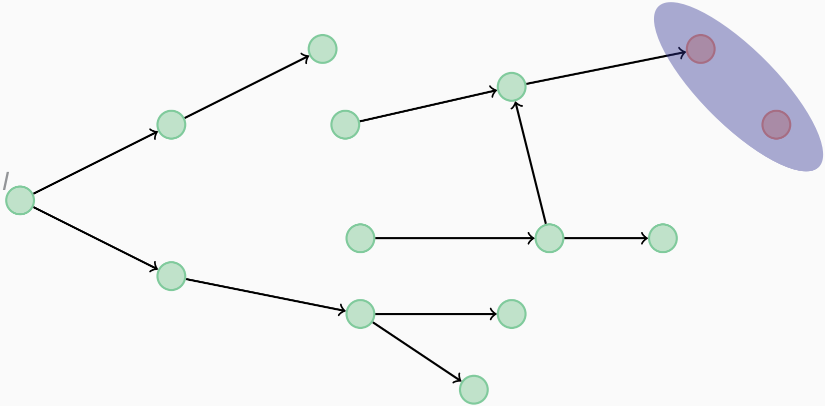

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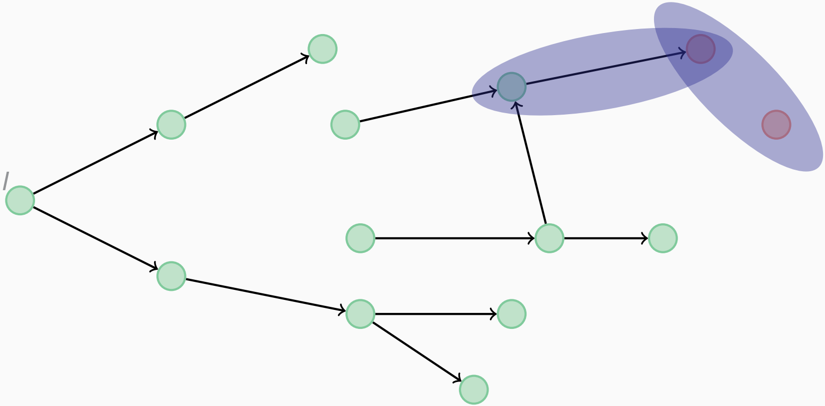

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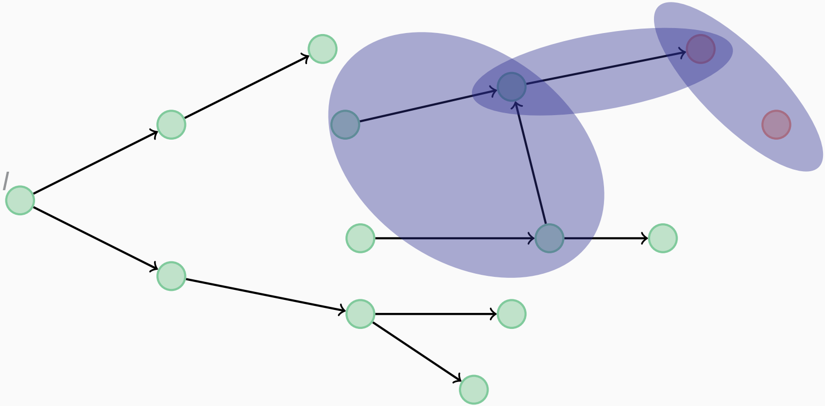

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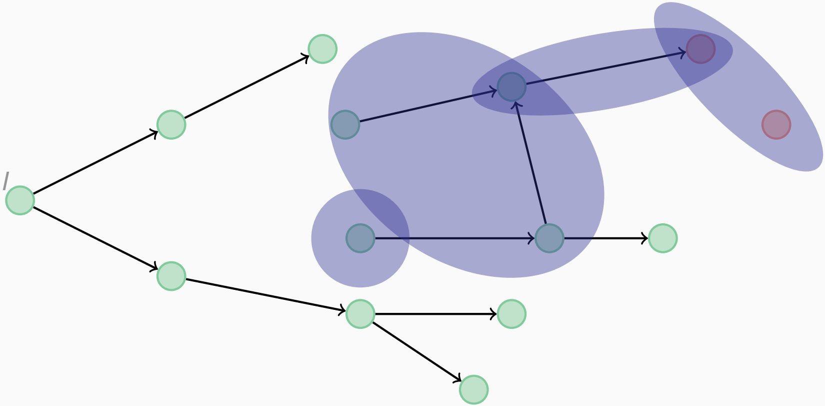

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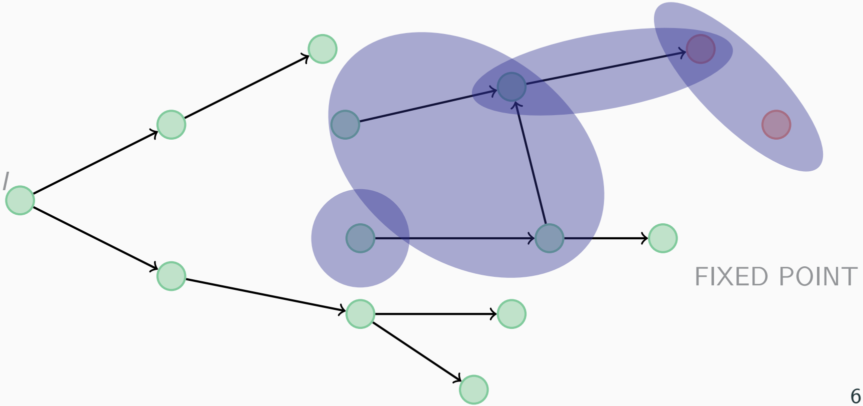

Personal view on OBDDs

BDDs are much more than a representation of a boolean formula.

Compressed truth tables: BDDs represent all the models of a boolean formula.

$\Rightarrow$ often too much large

Bounded Model Checking (for invariant properties)

At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.

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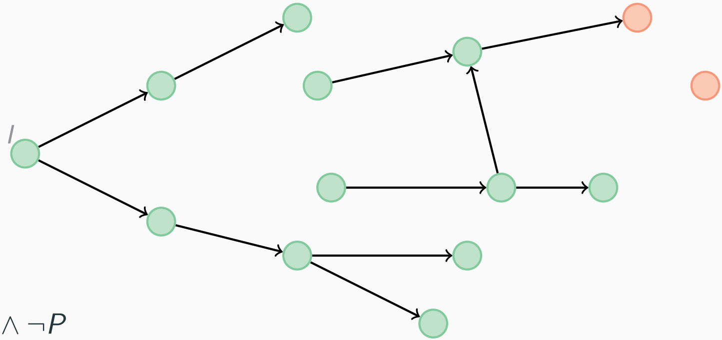

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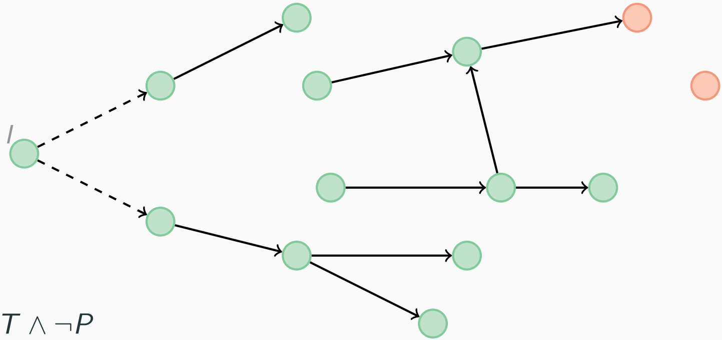

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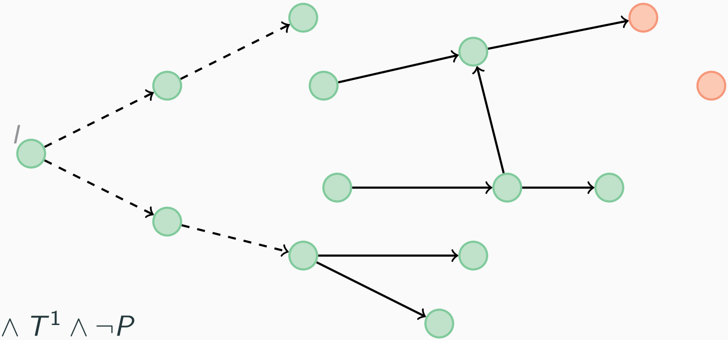

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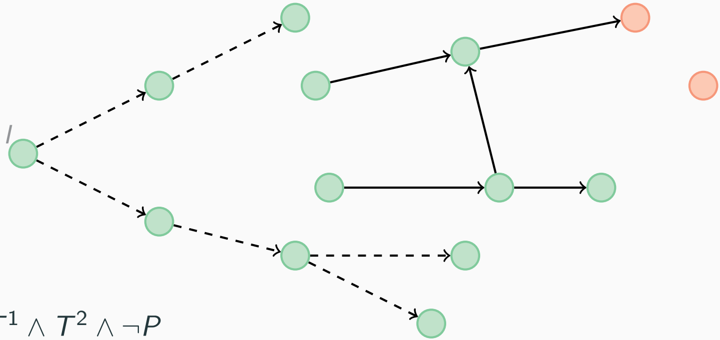

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- drawbacks:
  - in general it is not complete: we have to compute a big QBF to know the diameter of the graph;
  - it requires the unrolling of the transition relation:

$$I \wedge \left(T \wedge T^1 \wedge \cdots \wedge T^{k-1} \right) \wedge \neg P^k$$

    Both T and k can be very large: the formula can become too large for the SAT solver.

First Attempts to Incremental Inductive Verification

Inductive Verification

In order to prove that $P(x)$ is $\mathcal{M}$-invariant, one possibility is to check if P is inductive. With 2 SAT-solver calls, we check the validity of:

$$\begin{aligned} \text{(initiation)} \quad & I \Rightarrow P \\ \text{(consecution)} \quad & P \land T \Rightarrow P' \end{aligned}$$

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Why?

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- **Incremental proof**: look for a sequence of lemmata $\phi_1, \phi_2, \ldots, \phi_k = P$ such that ϕ_i is inductive relative to $\phi_1 \wedge \cdots \wedge \phi_{i-1}$, for all $1 < i \leq k$, *i.e.*,
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  It follows that $P \wedge \bigwedge_{i=1}^{k-1} \phi_i$ is an inductive strengthening.

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- both methods do *not* compute a formula R for the exact set of reachable states in $\mathcal{M}$;
- rather, they find a formula $F \wedge P$ that represents a larger set of states *all satisfying* $F \wedge P$:
  - $\Rightarrow$ this F is a much smaller formula than R.

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Naïve algorithm for finding an inductive strengthening:

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At the end, the inductive strengthening (if any) will be:

$$P \land \bigwedge_{err \in CTI} \neg err$$

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We establish the first inductive incremental lemma $\phi_1 := x \geq 0$:

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Now, $\phi_2 \coloneqq y \geq 1$ is inductive relative to ϕ_1:

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We have found the inductive strengthening $\phi_1 \land \phi_2$, by means of an incremental proof.

Limitation of incremental proofs

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Monolithic approach = worst case of incremental proofs.

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- “this algorithm is a result of asking the question: if the incremental method is often better for humans, might it be better for algorithms as well?" [Bra12];
- the core of the algorithm is the generalization an error state.

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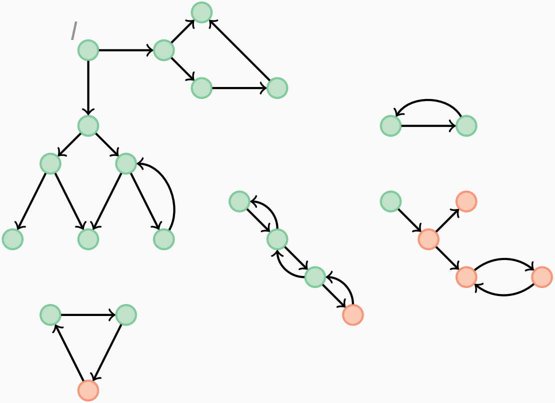

FSIS - Example - 1^{st} iteration

Check if P is inductive (relative to nobody). Check the validity of:

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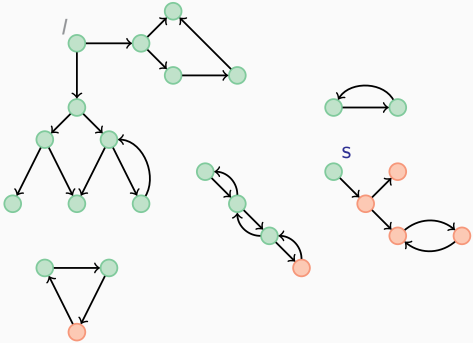
State s is a CTI.



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  - $\phi_1 \subseteq \neg s$; *(it excludes s)*
  - ϕ_1 is inductive (relative to its own); *(it includes at least all the reachable states)*
  - ϕ_1 is minimal. *(it excludes the maximal number of non-reachable states)*
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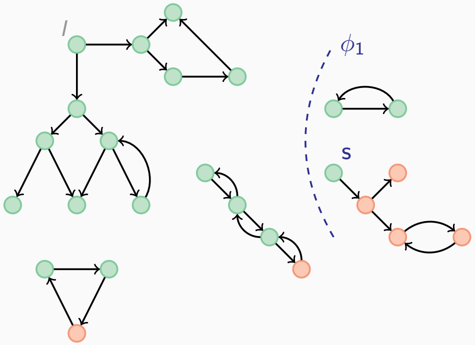
    - recall the nice property of clauses: if $c \subseteq d$ then $[\![c]\!] \subseteq [\![d]\!]$
- if ϕ_1 does exist, it becomes the first incremental lemma.

FSIS - Example - 1^{st} iteration

ϕ_1 can be thought as a "boolean" cutting plane.



Which states are excluded by ϕ_1? (i) those who can reach s
(ii) states “similar" to s (they share with s the dropped literals).

FSIS - Example - 2^{nd} iteration

Check if P is inductive relative to ϕ_1:

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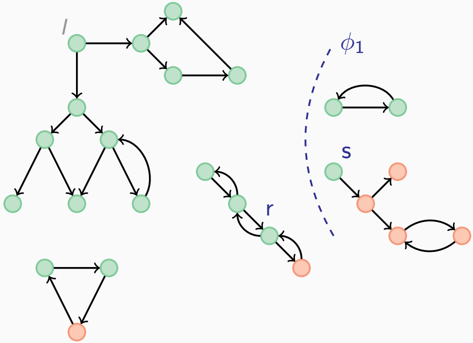
State r is a CTI.



FSIS - Example - 2^{nd} iteration

- generalization of error state r:
  - $\phi_2 \subseteq \neg r$;
  - ϕ_2 is inductive relative to ϕ_1;
  - ϕ_2 is minimal;
- ϕ_2 is the second incremental lemma.

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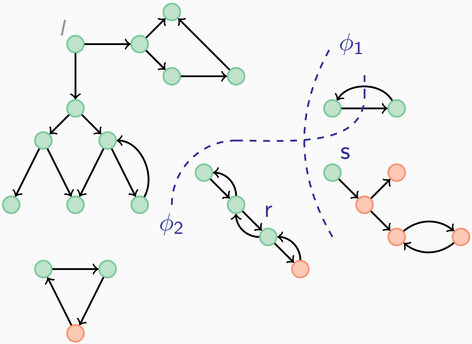
  - ϕ_2 is minimal;
- ϕ_2 is the second incremental lemma.



FSIS - Example - 2^{nd} iteration

Why "inductive relative to"?

- it would have been correct to generate ϕ_2 inductive (relative to its own), but it's more than what we need;
  - at the end we will consider the AND of all the lemmata;

FSIS - Example - 2^{nd} iteration

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- it would have been correct to generate ϕ_2 inductive (relative to its own), but it’s more than what we need;
  - at the end we will consider the AND of all the lemmata;
- in general, it is faster to generate “inductive relative to" clauses.
  - intuitively, we are considering many fewer states of the system.

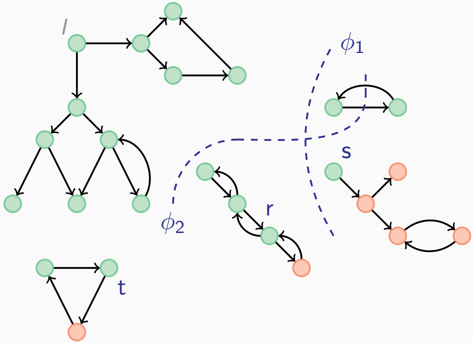
FSIS - Example - 3^{rd} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge P \wedge T \not\Rightarrow P'$

State t is a CTI.



FSIS - Example - 3^{rd} iteration

- generalization of error state t:
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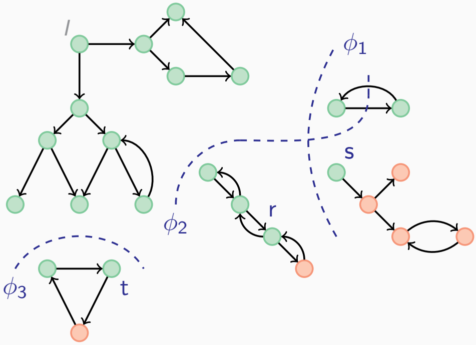

FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P \wedge T \Rightarrow P'$

FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P \wedge T \Rightarrow P'$

- $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P$ is an inductive strengthening.
- P is $\mathcal{M}$-invariant.

Worst case

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- suppose that an error state s does *not* have a minimal inductive generalization;
- worst case: we proceed with the monolithic technique;

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- eventually,
  - either $I \land \neg P$ is SAT: P is not invariant;
  - or we find an inductive strengthening $P \land \bigwedge_{i=0}^{n} \phi_i$;

Complexity and Parallelization

Complexity:

- it is on the convergence of the procedure, not on the calls to the SAT-solver as before;
- each SAT-solver call is relatively small compared to those made by BMC.

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Parallelization:

- straightforward; "by simply using a randomized decision procedure to obtain the CTIs, each process is likely to analyze a different part of the state-space." [BM07]

IC3

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- FSIS sometimes enters a long search for the next relatively inductive clauses;
- IC3 de-emphasized global information in favor of stepwise information: we will generate clauses that ensure that an error is unreachable up to some number of steps.

IC3 - Data Structures

Sequence of frames $F_0(= I), F_1, F_2, \ldots, F_k$:

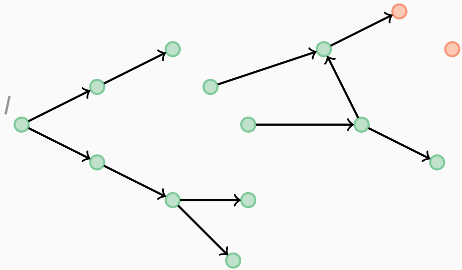
- each F_i is an over-approximation of the set of states reachable in at most k steps;
- each F_i is a set of clauses, *i.e.*, a CNF formula;
- the algorithm stops when $F_i \equiv F_{i+1}$. We will maintain the invariant that $clauses(F_{i+1}) \subseteq clauses(F_i)$: the equivalence check is simply a sintactic test: $F_i = F_{i+1}$.

IC3 - 1^{st} iteration

Check if there are counterexamples of length 0 or 1 with these two SAT-queries:

✗ $I \wedge \neg P$

✗ $F_0(= I) \wedge T \wedge \neg P'$



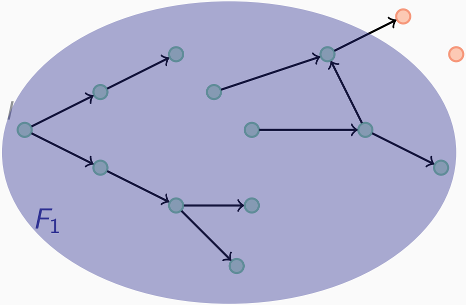
IC3 - 1^{st} iteration

Check if there are counterexamples of length 0 or 1 with these two SAT-queries:

✗ $I \wedge \neg P$

✗ $F_0 (= I) \wedge T \wedge \neg P'$

Since $F_0 \wedge T \Rightarrow P'$, we set $F_1 := P$. (over-approximation)



IC3 - 2^{nd} iteration

At iteration k, check if $F_k \wedge T \wedge \neg P'$; in this case $(k = 1)$:

$$\checkmark \quad F_1 \wedge T \wedge \neg P'$$

i.e., there exists an F_k-state that leads in one step to an error state?

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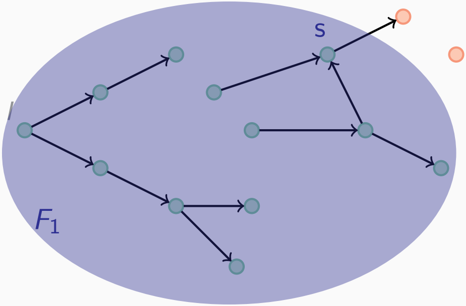

IC3 - 2^{nd} iteration (blocking phase)

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$\Rightarrow \phi_1$ excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

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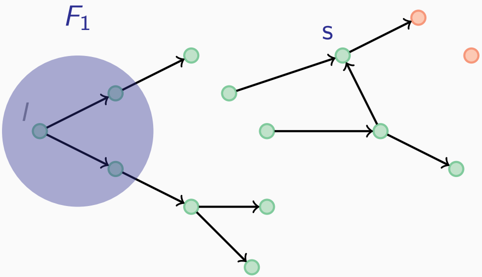
$\Rightarrow$ ϕ_1 excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

We add ϕ_1 to all the previous frames. In this case $F_1 := F_1 \wedge \phi_1$.



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We have found a CTI s such that $s \models F_k \wedge T \wedge \neg P'$.

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if $\neg s$ is inductive relative to F_{k-1}, then generate a minimal subclause $c \subseteq \neg s$ inductive relative to F_{k-1}, *i.e.*, c holds for at least all states reachable in i steps.

$\Rightarrow$ add c to frames $F_0 \ldots F_{k+1}$, *i.e.*, refine the over-approximations.

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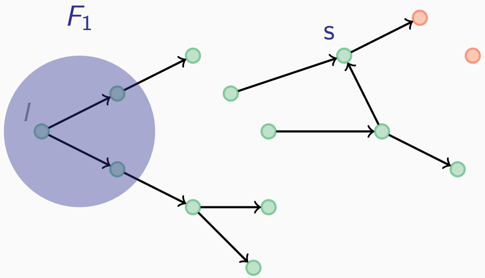
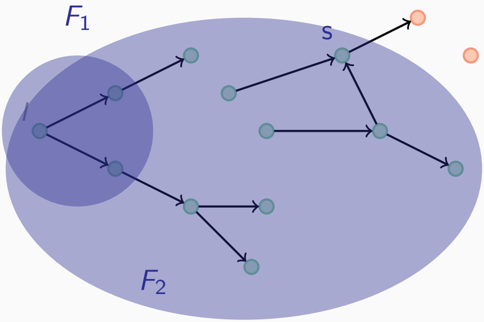
We create a new frame only when $F_k \wedge T \Rightarrow P'$ is valid.



IC3 - 2^{nd} iteration

We create a new frame only when $F_k \wedge T \Rightarrow P'$ is valid.

In this case, $F_1 \wedge T \Rightarrow P'$ is valid. We create a new frame $F_2 := P$.



IC3 - Propagation phase

Propagation phase: After creating a new frame $F_{k+1} := P$, we perform the propagation phase: we push forward the clause discovered in frame F_i for some i.

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For all $0 \leq i \leq k$ and $c \in F_i$, check if

$$F_i \wedge T \Rightarrow c'$$

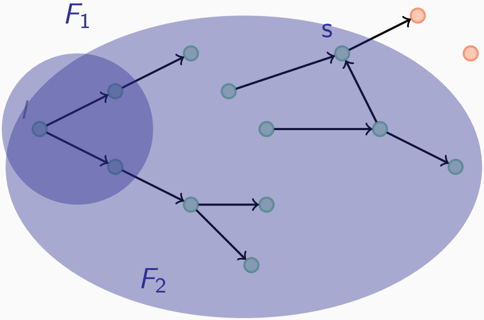
If $c \notin \mathit{clauses}(F_{i+1})$, then set $F_{i+1} \coloneqq F_{i+1} \cup \{c\}$

$\Rightarrow$ it propagates forward the errors

$\Rightarrow$ it helps the discovery of mutually inductive clauses

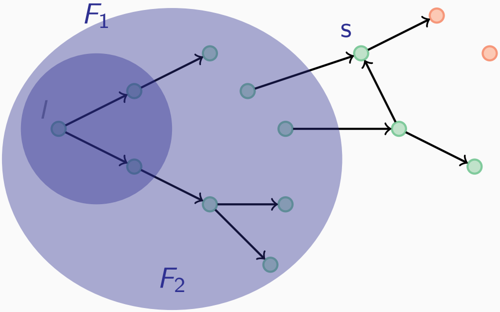
IC3 - 3^{rd} iteration

Check if $F_2 \wedge T \wedge \neg P'$ (✓). $\neg s$ is inductive relative to F_1: error state s is not reachable for at least $k = 2$ steps.



IC3 - 3^{rd} iteration

Check if $F_2 \wedge T \wedge \neg P'$ (✓). $\neg s$ is inductive relative to F_1: error state s is not reachable for at least $k = 2$ steps. Blocking phase: find minimal subclause $\phi_2 \subseteq \neg s$ inductive relative to F_1. Add ϕ_2 to frames F_0 and F_1.



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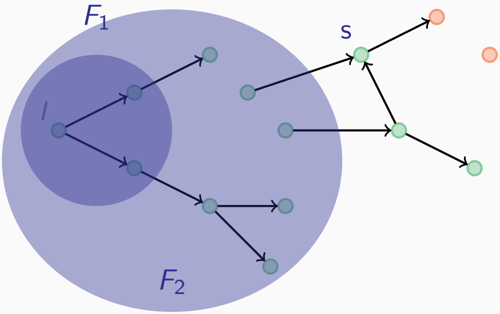

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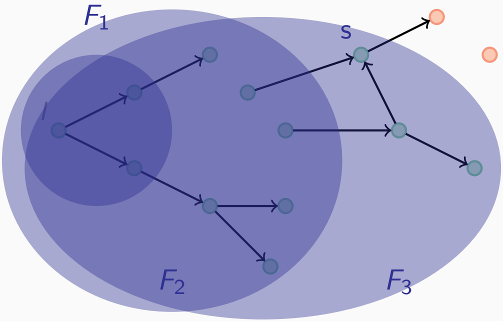

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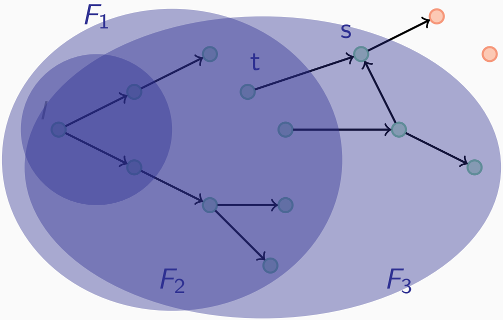
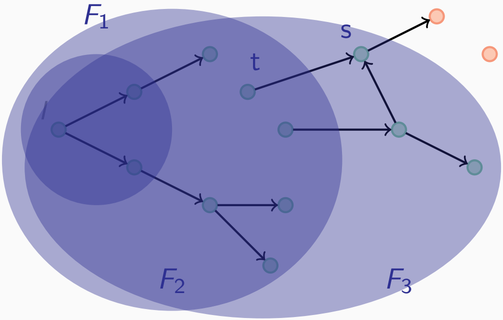


IC3 - 4^{rd} iteration

Again $F_3 \wedge T \wedge \neg P'$ (✓). But now $\neg s$ is not inductive relative to F_2: error state s could be reachable in $k = 3$ steps ...



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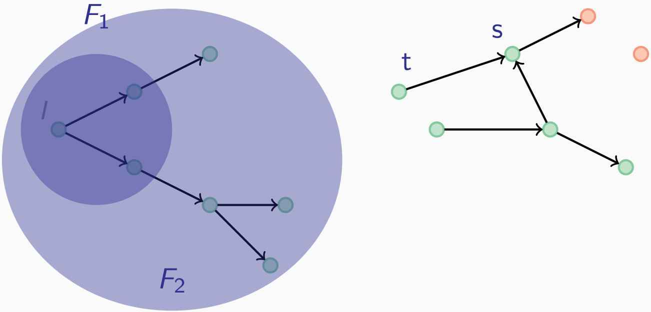
"t is the new s" ;-)

IC3 - Recursion

We want to remove error state t from F_2. $\neg t$ is inductive relative to F_1: find min subclause $\phi_4 \subseteq \neg t$ and add it to F_0,F_1,F_2.



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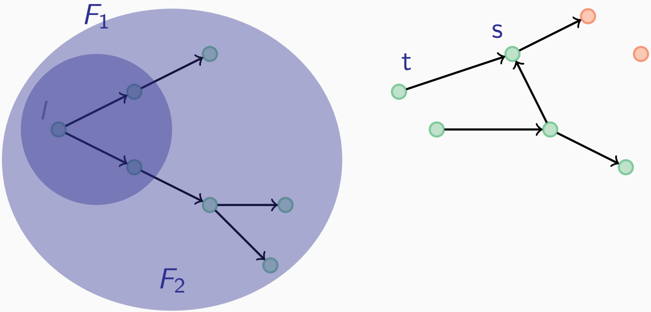
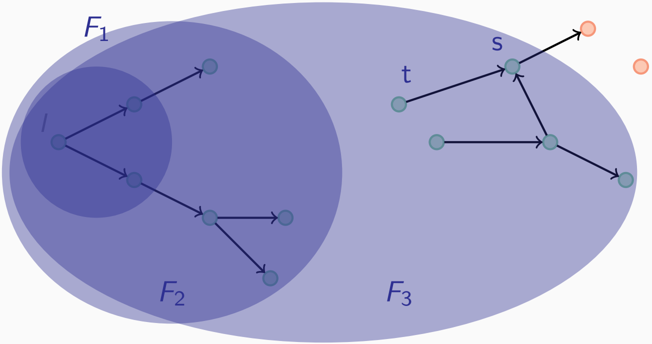


If in this process we had gone back with recursion until an initial state, then we would have found a counterexample.

IC3 - Termination

Now error state s in frame F_3 can be generalized: find min clause $\phi_5 \subseteq \neg s$ inductive relative to F_2.



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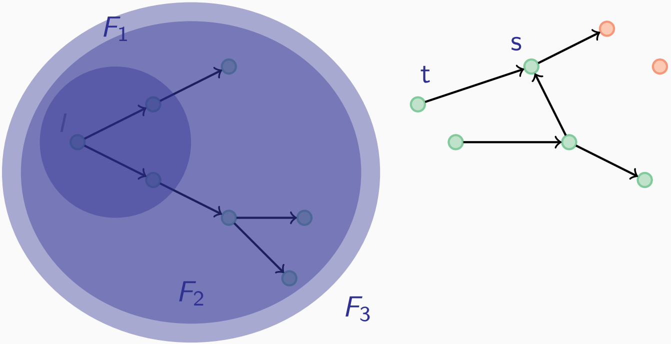

$F_2 = F_3$: IC3 terminates with True.

Beyond IC3

- FAIR: IC3 for ω-regular properties (*e.g.*, LTL) [Bra+11];
- IICTL: IC3 for CTL properties [HBS12];
- Infinite-state: software model checking via IC3 [CG12].

Thanks for the attention...

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