Parity-energy ATL for Qualitative and Quantitative Reasoning in MAS

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Udine, February 15, 2019

Outline

1. Introduction and motivations
2. The logic pe-ATL
   - pe-ATL at work
3. Model checking pe-ATL
   - Warming up: Parity and energy conditions in isolation
   - Unbounded $[-\infty, +\infty]$ and bounded $[a, b]$ energy range
   - Left-bounded $[a, +\infty]$ and right-bounded $[-\infty, b]$ energy range
4. Conclusions

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- Intelligent (take decisions, moves)
- Independent
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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

Theorem (Goranko, TARK 2001)

CL can be embedded into ATL

ATL: syntax and models

- **Syntax.** Formulae of ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A \rangle\rangle \bigcirc \varphi \mid \langle\langle A \rangle\rangle \Box \varphi \mid \langle\langle A \rangle\rangle \varphi \mathcal{U} \varphi$$

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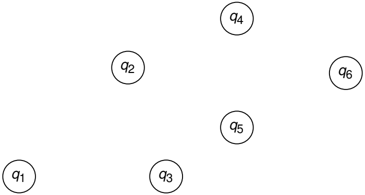
- **Models.** CGS's (concurrent game structure) are labeled transition systems:



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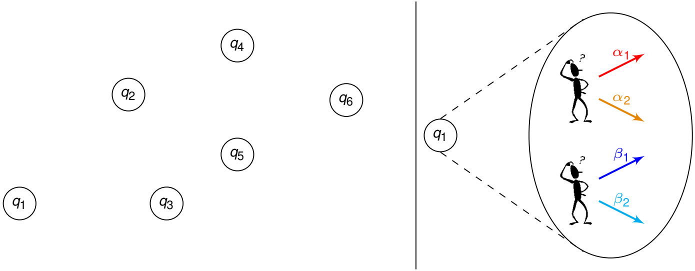

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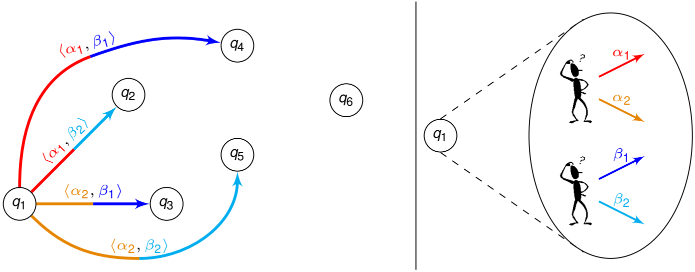

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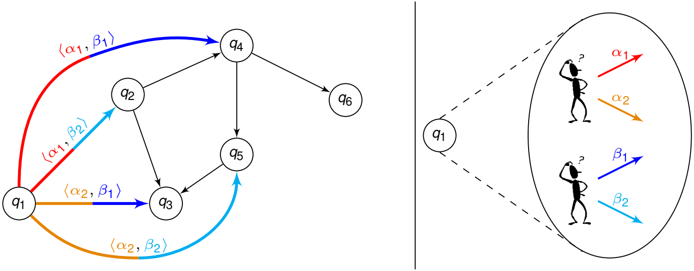

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ATL: (intuitive) semantics

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- ATL = coalition abilities + temporal goals
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Sample scenario:

- printing system: n printers + shared bounded printing queue
- $n + m$ agents (n printers + m users/environment)
- printer actions: { **n** (*do-nothing*), **p** (*print*) }
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pe-ATL abilities

- avoid errors (*i* printers do *print* and queue only contains $j < i$ jobs) (safety $\mapsto$ coalition+temporal)
- queue is emptied infinitely often (Büchi $\mapsto$ parity)
- users send infinitely many jobs $\Rightarrow$ queue is filled up infinitely often (fairness $\mapsto$ parity)
- devices' turnover (alternation $\mapsto$ energy)

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pe-ATL abilities

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  can be expressed in standard ATL
  (safety $\mapsto$ coalition+temporal)
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Strategies

- history: finite path over the CGS (game arena)

$$q_0 \xrightarrow{\alpha_1} q_1 \xrightarrow{\alpha_2} q_2 \xrightarrow{\alpha_3} \ldots \xrightarrow{\alpha_k} q_k$$

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- strategy for agent a: a function that chooses the next move of agent a based on the history of the system

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- strategy for agent *a*: a function that chooses the next move of agent *a* based on the history of the system
- strategy for team *A*: a function that chooses the next move for each agent in *A* based on the history of the system

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A strategy for proponent team induces an infinite tree over the CGS

Different branches correspond to different choices of the opponent

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A strategy for proponent team induces an infinite tree over the CGS

Different branches correspond to different choices of the opponent

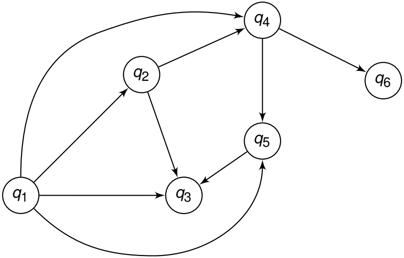
- memoryless strategy: only consider the last state of the history q_k (current state)
- uniform strategy: only consider the last state of the history q_k (current state) and the current energy level

Parity and energy conditions

- parity condition:
  - nodes of the graph are assigned with a natural number (color, parity)
  - an infinite path over the graph satisfies the parity condition if the smallest parity occurring infinitely often is even
- energy condition:
  - an energy range, an initial energy level, and weights on transitions are given (all rational numbers)
  - energy value evolves along a path over the graph according to the initial energy level and to transition weights
  - an infinite path over the graph satisfies the energy condition if the energy values stays in the range

pe-ATL: syntax and models

- **Syntax.** The same as ATL
- **Models.** pe-CGS = CGS + parity + energy conditions



G

- **vertices** labeled by **atomic propositions**
- in vertices agents choose **actions**
- possible combinations $\rightarrow$ **transitions** (edges of the graph)
- parity condition
- energy condition

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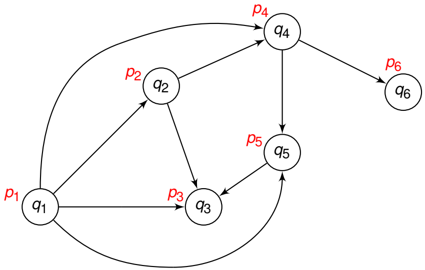

$\langle G, p \quad \rangle$

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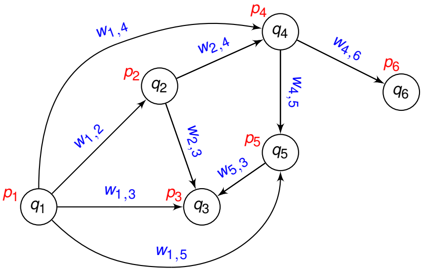

$\langle G, p, e \rangle$

- initial energy level $\mathcal{E}_0$
- energy range $[a, b]$

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pe-ATL: (intuitive) semantics

Collective (p, e)-strategy for the proponent team to guarantee φ holds

$\langle\langle A \rangle\rangle \bigcirc \varphi$	next
$\langle\langle A \rangle\rangle \Box \varphi$	always
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regardless of actions performed by other agents (opponent)

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strategies must be (p, e)-strategies, i.e.,
they only produce plays satisfying parity and energy conditions

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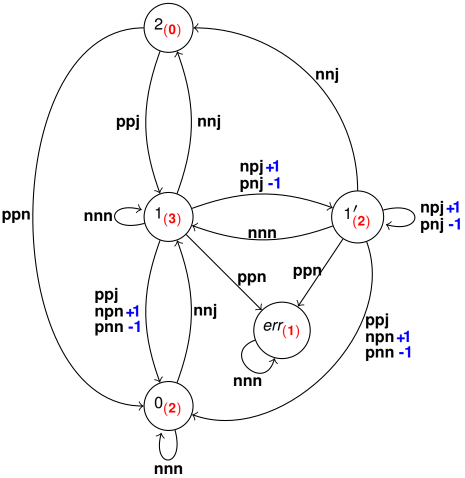

agents $= \{p_1, p_2, u\}$

actions

	p_1	p_2	u	*joint actions*
0	**n**	**n**	**nj**	{**nnn**, **nnj**}
1	**np**	**np**	**nj**	{**nnn**, **nnj**, **npn**, **npj**, **pnn**, **pnj**, **ppn**, **ppj**}
1′	**np**	**np**	**nj**	{**nnn**, **nnj**, **npn**, **npj**, **pnn**, **pnj**, **ppn**, **ppj**}
2	**p**	**p**	**nj**	{**ppn**, **ppj**}
err	**n**	**n**	**n**	{**nnn**}

energy weights

$$w(\mathbf{nn}x) = w(\mathbf{pp}x) = 0$$
$$w(\mathbf{np}x) = +1$$
$$w(\mathbf{pn}x) = -1$$

energy range = [0,1]

initial energy level $\mathcal{E}_0 = 0$

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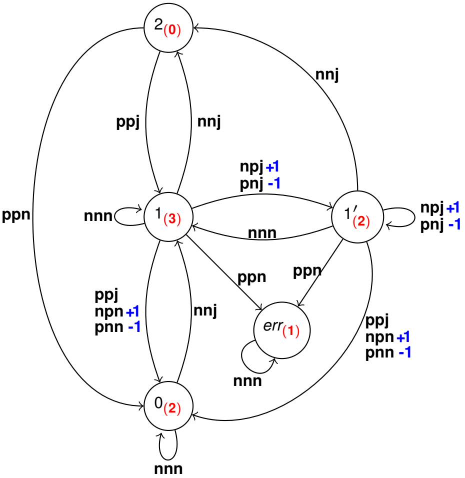

$$\mathcal{G}, 0 \models \langle\langle\{p_1, p_2\}\rangle\rangle\Box\neg err$$

$\exists$ joint strategy for p_1 and p_2 s.t.:

- error state is avoided (temporal)
- all jobs are processed (parity)
- printers alternate (energy)

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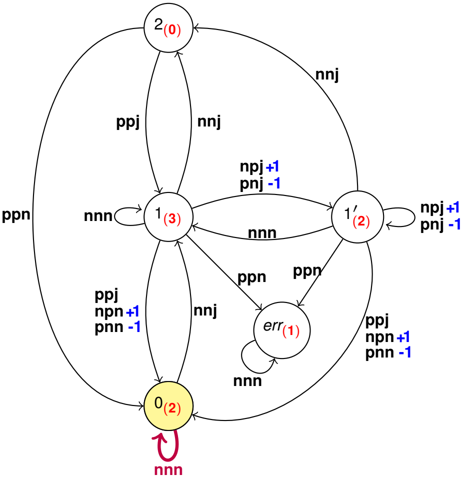

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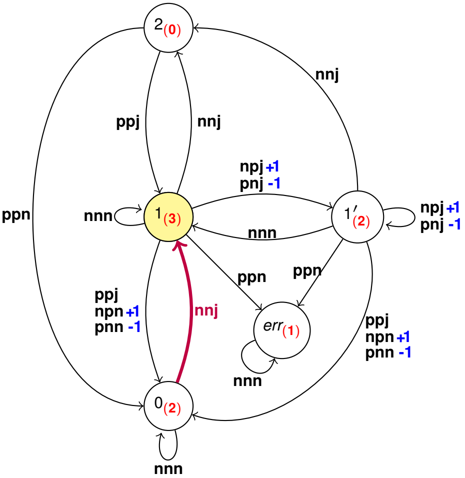

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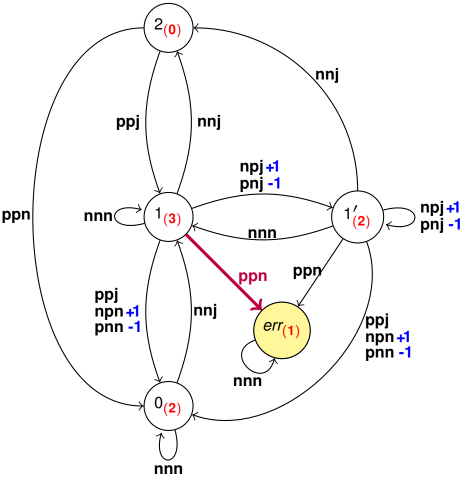

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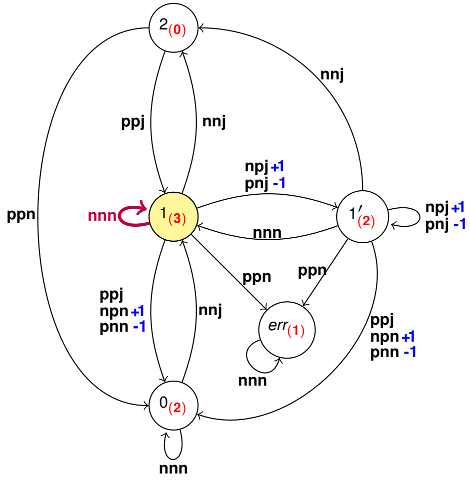

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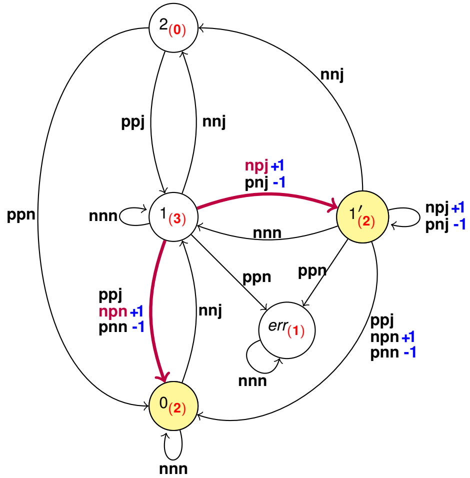

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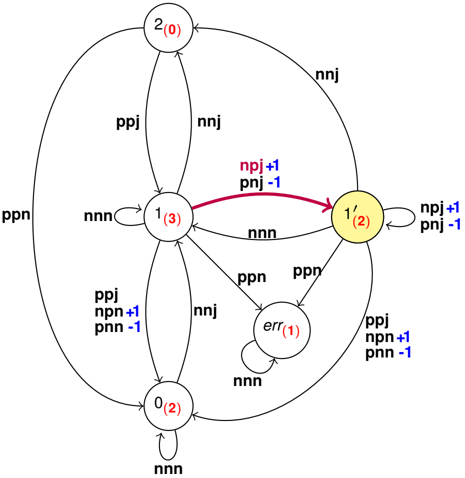

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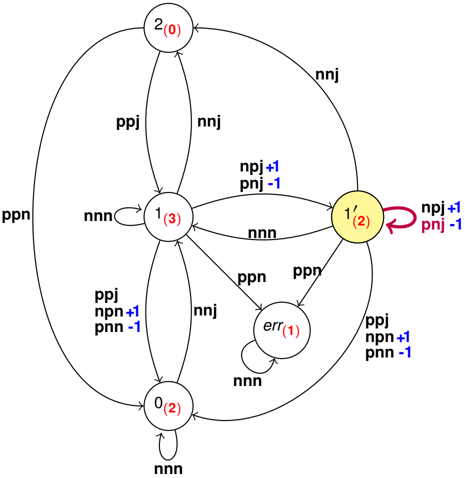

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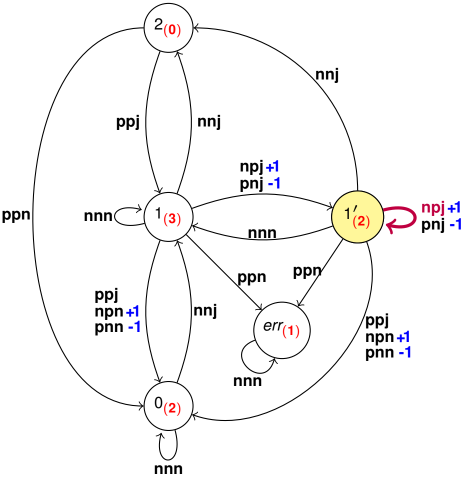

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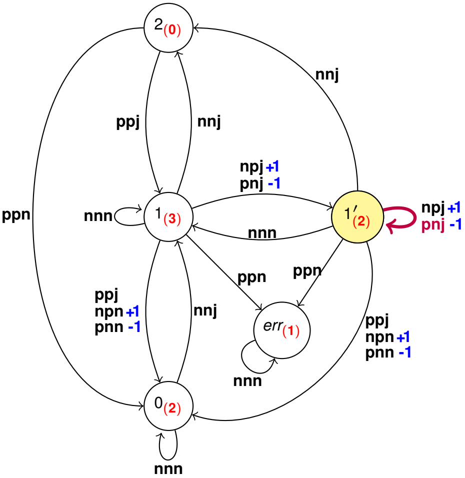

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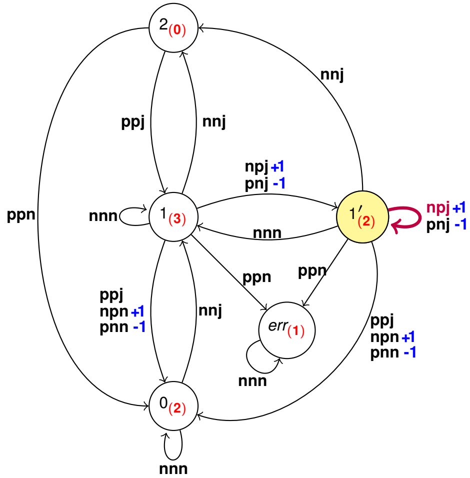

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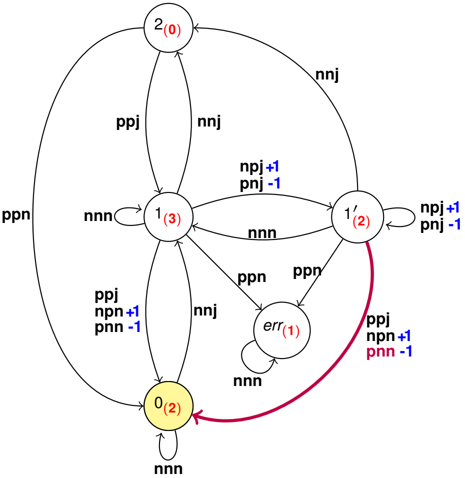

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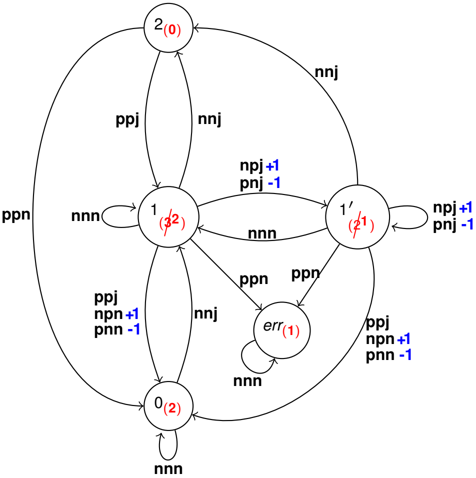

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- if user sends infinitely many jobs, then queue is filled up infinitely often (parity)
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The model checking problem

Definition (pe-ATL model checking problem)

Given a pe-CGS $\mathcal{G} = \langle G, p, e \rangle$ and a pe-ATL formula φ, establish whether $\mathcal{G} \models \varphi$

We consider the following cases:

- unbounded energy range $[-\infty, +\infty]$ NP
- bounded energy range $[a, b] \in \mathbb{Q}$ NEXPTIME
- left-bounded energy range $[a, +\infty]$ NP
  (right-bounded is symmetric)

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Parity and energy conditions in isolation

Definition (p-ATL/e-ATL)

- p-ATL: relax the energy condition
  strategies fulfill parity condition only
- e-ATL relaxing the parity condition
  strategies fulfill energy condition only

Lemma

p-ATL/e-ATL model checking problem easily reduce to pe-ATL one

- spurious parity condition: $p(q) = 0$ for all $q \in Q$
  all parity are even and so is the smallest occurring infinitely often
- spurious energy condition: weights, initial energy level, and energy bounds are set to 0
  initial energy level is in range and never changes

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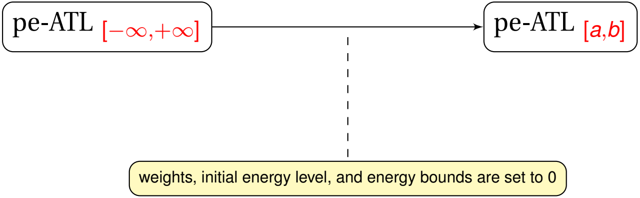
Unbounded energy range $[-\infty, +\infty]$

- Reduction to p-ATL (just ignore the energy condition)
- Reduction to the case of bounded energy range $[a, b]$



Bounded energy range $[a, b]$

- $a \neq -\infty$, $b \neq +\infty$

Lemma (normalization)

It is possible to focus on instances where no rationals are involved

- integer energy range ($a, b \in \mathbb{Z}$)
- integer initial energy level ($\mathcal{E}^{init} \in \mathbb{Z}$)
- weights over transitions are integers as well

Lemma (positional strategies)

- a (p, e)-strategy exists iff a uniform one exists (bounded instance)
- a (p, e)-strategy exists iff a memoryless one exists (unbounded instance)

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(Un)Bounded energy range $[a, b]$: Complexity

- uniform strategies are positional in $Q \times [a, b]$
  - exponentially many positions (q, *energy-level*) when a and b are in binary—thanks to normalization
- memoryless strategies are positional in Q
  - polynomially many positions q

A non-deterministic algorithm:

- guess the strategy
- return *false* when a loop with odd parity or an out-of-range is detected
- no position is visited twice
- bounded case: exponential time
- unbounded case: polynomial time

Outline

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2. The logic pe-ATL
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3. Model checking pe-ATL
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   - Left-bounded $[a, +\infty]$ and right-bounded $[-\infty, b]$ energy range
4. Conclusions

Left-bounded energy range $[a, +\infty]$

(right-bounded energy range $[-\infty, b]$ is symmetric)

- Model-theoretic argument (technically quite involved)
- Difficulty: the space of positions (q, *energy*-*level*) is infinite
- We define suitable structures (witnesses)
  - compact representations for strategies
  - bounded size
  - we prove it to be complete for strategies
- A non-deterministic algorithm guesses one such structure and check that it is indeed a witness for the desired strategy

Key ideas

- A witness (for a $\langle\langle A\rangle\rangle\Box\psi$ formula) is a pair of graphs

$$(S_1, S_2)$$

- Elements of such graphs are positions (q, *energy-level*)

$(q, \textit{energy-level}) \in S$ iff there is a *winning* strategy for A, i.e., a (p, e)-strategy that guarantees the invariant ψ

- Left-bounded range ensures monotonicity

a strategy exists from $(q, \textit{energy-level})$ iff a strategy exists from (q, E) for all $E \geq \textit{energy-level}$

- Thus, only the smallest energy level appears in S_1 and S_2 for each q

$$|S_1| \leq |Q|, \qquad |S_2| \leq |Q|$$

- S_1 represents the strategy for parity and temporal goals
  S_2 contains increasing loops to increase the energy levels

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Alechina, Logan, Nguyen, Raimondi, JCSS (2017)

Model-checking for Resource-Bounded ATL with production and consumption of resources, p. 126–144

From witnesses to strategies

- *internal constraints*
  - e.g., elements of S_1 and S_2 satisfy the invariant ψ in a formula $\langle\langle A\rangle\rangle\Box\psi$
- *diagonal constraints*
  - e.g., elements of S_1 with low energy level also occur as (and can be merged with) elements of S_2
- the unfolding/merging of S_1 and S_2 corresponds to the outcome of a winning strategy for A

From strategies to witnesses

Witness construction
(from the tree $\mathcal{T}$ of outcomes of a winning strategy for A)

- q appears in the witness iff it appears in the tree $\mathcal{T}$
- suitably cut tree $\mathcal{T}$ into a finite (not bounded) prefix
- for every q, a representative node in the cut of $\mathcal{T}$ is chosen
  - based on their topological order and their energy level in the tree
- energy level and outgoing transition for q in the witness are determined by its representative in the cut of $\mathcal{T}$

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4. **Conclusions**

Conclusions

- pe-ATL: coalitional abilities to pursue temporal goals while satisfying qualitative (parity) and quantitative (energy) conditions
- pe-ATL model checking problem

Theorem

The model checking problem for pe-ATL is:

- in NEXPTIME if the energy range is bounded ($[a, b]$)
- in NPTIME if the energy range is unbounded ($[-\infty, +\infty]$)
- in NPTIME if the energy range is left- or right-unbounded ($[a, +\infty]$ or $[-\infty, b]$)

Notice that ATL* is 2EXPTIME-complete

Future work

- to establish thigh complexity bounds (parity game complexity)
- to extend the proposed framework to ATL*
  - comparison of the expressive power with ATL* and other logics for strategic reasoning, e.g., Strategy Logic (SL)
- different modeling choices
  - energy level evolves along the entire game
  - limit opponent power with parity and energy conditions as well
  - multiple quantitative dimension (several resources besides energy)

The end

Thank you!